11-1 Additional Vocabulary Support
Space Figures and Cross Sections

Concept List

- cross section
- face
- polyhedra
- edge
- horizontal plane
- vertex
- Euler’s Formula
- net
- vertical plane

Choose the concept from the list above that best represents the item in each box.

1. polyhedra
2. $F + V = E + 2$
   - Euler’s Formula
3. net
4. $BF$
   - edge
5. cross section
6. $A$
   - vertex
7. quadrilateral $ABCD$
8. plane $J$
   - horizontal plane
9. plane $K$
   - vertical plane
Think About a Plan
Space Figures and Cross Sections

Visualization  Draw and describe a cross section formed by a plane intersecting the cube as follows.

The plane is tilted and intersects the front and back faces of the cube perpendicular to the left and right faces.

Understanding the Problem

1. What is a cube? Draw a typical view of a cube and describe a cube.
   a rectangular prism with six square faces, all congruent

2. What is a plane? Draw a typical view of a plane and describe a plane.
   a two-dimensional surface unbounded in all directions

3. What is a cross section? Draw a plane passing through your cube that is parallel to the top and bottom faces of your cube. Explain why the cross section is a square. A cross section is an intersection of a solid and a plane. It is a very thin slice of the solid. Because the plane is parallel to two opposing faces, the cross section is congruent to the faces. So, the cross section is a square congruent to the faces.

Planning the Solution

4. How can you use your understanding of cubes, planes, and cross sections to draw a plane that is tilted and intersects the front and back faces of the cube perpendicular to the left and right faces?
   Look at the drawing of a parallel plane to understand how a tilted plane is different.

5. You showed that if a plane is not tilted, the cross section is a square. How can you use this knowledge to predict what the cross section will look like if the plane is tilted? Contrast the cross section you get when the plane is parallel to the cross section you will get when the plane is tilted.

Getting an Answer

6. Prepare to draw a plane that is tilted and intersects the front and back faces of the cube perpendicular to the left and right faces by studying your drawing of a plane parallel to the top and bottom faces. Imagine the thin slice created when this tilted plane passes through the cube. Describe this cross section.
   The cross section is a rectangle. Two sides are the same length as the edges of the cube, and two sides are longer than the edges.
For each polyhedron, how many vertices, edges, and faces are there? List them.

1. 6 vertices: \(A, B, C, D, E, P\); 10 edges: \(AP, BP, CP, DP, EP, AB, BC, CD, DE, EA\); 6 faces: \(\triangle APB, \triangle BCP, \triangle CDP, \triangle DEP, \triangle EAP, \) and pentagon \(ABCDE\)

2. 8 vertices: \(A, B, C, D, E, F, G, H\); 12 edges: \(AB, BC, CD, DA, AE, BF, CG, DH, EF, FG, EH, GH\); 6 faces: quadrilaterals \(ABCD, ABFE, CBFG, DCGH, ADHE, EFGH\)

For each polyhedron, use Euler's Formula to find the missing number.

3. Faces: 6  Edges: 12  Vertices: 8
5. Faces: 10  Edges: 18  Vertices: 10
6. Faces: 10  Edges: 24  Vertices: 16
7. Faces: 8  Edges: 12  Vertices: 6

Verify Euler's Formula for each polyhedron. Then draw a net for the figure and verify Euler's Formula for the two-dimensional figure.

8. polyhedron: 7 faces, 15 edges, 10 vertices; \(F + V = E + 2, \) \(7 + 10 = 15 + 2, \) \(17 = 17\); net: 7 faces, 24 edges, 18 vertices; \(F + V = E + 1, \) \(7 + 18 = 24 + 1, \) \(25 = 25\)

9. polyhedron: 6 faces, 12 edges, 8 vertices; \(F + V = E + 2, \) \(6 + 8 = 12 + 2, \) \(14 = 14\); net: 6 faces, 19 edges, 14 vertices; \(F + V = E + 1, \) \(6 + 14 = 19 + 1, \) \(20 = 20\)

Describe each cross section.

10. a circle
11. a hexagon
12. a parallelogram
13. **Open-Ended** Sketch a polyhedron with more than four faces whose faces are all triangles. Label the lengths of its edges. Use graph paper to draw a net of the polyhedron.  

*Answers may vary. Sample drawing is at the right.*  

Use Euler’s Formula to find the number of vertices in each polyhedron.

14. 6 faces that are all parallelograms 8

15. 2 faces that are heptagons, 7 rectangular faces 14

16. 6 triangular faces 5

**Reasoning** Can you find a cross section of a square pyramid that forms the figure? Draw the cross section if the cross section exists. If not, explain.

17. square

18. isosceles triangle

19. rectangle that is not a square  

*No; any plane that intersects the four faces has either a trapezoid, a square, or no parallel sides in its cross section.*

20. equilateral triangle

21. scalene triangle

22. trapezoid

23. What is the cross section formed by a plane containing a vertical line of symmetry for the figure at the right? **triangle**

24. What is the cross section formed by a plane that is parallel to the base of the figure at the right? **octagon**

25. **Reasoning** Can a polyhedron have 19 faces, 34 edges, and 18 vertices? Explain.  

*No; by Euler’s Formula, there should be 35 edges, 18 faces, or 17 vertices.*


*No; all of the faces of a polyhedron are polygons. A cone has no faces that are polygons.*

27. **Visualization** What is the cross section formed by a plane that intersects the front, right, top, and bottom faces of a cube? **a rectangle or a trapezoid**
For each polyhedron, how many vertices, edges, and faces are there? List them.

1. Vertices: A, B, C, D, E, F, G, H; 8
   Edges: AB, AF, AD, BE, BC, CD, CH, HE, HG, GF, EF, DG; 12
   Faces: ABCD, ABFE, ADGF, DCHG, BCHE, EFGH; 6

2. Vertices: V, W, X, Y, Z; 5
   Edges: XY, XW, XZ, VX, VW, VZ, WZ; 8
   Faces: VWZ, XYZ, WXV, XWZ, YVZ; 5

For each polyhedron, use Euler’s Formula to find the missing number.

3. Faces: [ ] Edges: 8 Vertices: 5
   To start, use Euler’s formula, then identify $F + V = E + 2$
   
4. Faces: 6 Edges: [ ] Vertices: 8
5. Faces: 4 Edges: 6 Vertices: [ ]

Verify Euler’s Formula for each polyhedron. Then draw a net for the figure and verify Euler’s Formula for the two-dimensional figure.

6. 8 faces, 18 edges, 12 vertices; $8 + 12 = 18 + 2$
   4 faces, 6 edges, 4 vertices; $4 + 4 = 6 + 2$
   8 faces, 29 edges, 22 vertices; $8 + 22 = 29 + 1$
   4 faces, 9 edges, 6 vertices; $4 + 6 = 9 + 1$
   Use Euler’s Formula to find the number of vertices in each polyhedron.

8. 6 faces that are all squares
9. 1 face that is a hexagon, 6 triangular faces
10. 2 faces that are pentagons, 5 rectangular faces

11. **Reasoning** Can a polyhedron have 20 faces, 30 edges, and 13 vertices?
    Explain. **No; by Euler’s Formula, there should be 31 edges, 19 faces, or 12 vertices.**

    **No; all of the faces of a polyhedron are polygons. A cylinder has faces that are circles, and circles are not polygons.**
Describe each cross section.

13. To start, visualize the plane’s intersection with the solid.

14. a triangle

15. an oval (ellipse)

Reasoning  Can you find a cross section of a square pyramid that forms the figure? Draw the cross section if the cross section exists. If not, explain.

16. isosceles triangle

17. trapezoid

18. scalene triangle

19. square

20. What is the cross section formed by a plane containing a vertical line of symmetry for the figure at the right? rectangle

21. What is the cross section formed by a plane that is parallel to the base of the figure at the right? hexagon
Multiple Choice

For Exercises 1–5, choose the correct letter.

1. A polyhedron has 6 vertices and 9 edges. How many faces does it have?  B
   - A 3
   - B 5
   - C 7
   - D 9

2. A polyhedron has 25 faces and 36 edges. How many vertices does it have?  H
   - F 11
   - G 12
   - H 13
   - I 14

3. Which of the following shows a net for a solid that has 8 faces, 12 vertices, and 18 edges?  D
   - A
   - B
   - C
   - D

4. What is the cross section formed by a plane that contains a vertical line of symmetry for a tetrahedron?  F
   - F triangle
   - G square
   - H rectangle
   - I pentagon

5. What is the cross section formed by a plane that intersects three faces of a cube?  A
   - A triangle
   - B square
   - C rectangle
   - D pentagon

Short Response

6. How many edges and vertices are there for an octahedron, a polyhedron with eight congruent triangular faces?
   - [2] 12 edges AND 6 vertices
   - [1] 12 edges OR 6 vertices
   - [0] no correct response given
11-1 Enrichment

Space Figures and Cross Sections

Archimedean Solids

An Archimedean solid is a polyhedron that has a similar arrangement of polygons about each vertex. The faces of an Archimedean solid are made up of two or more types of regular polygons.

In total, there are 13 Archimedean solids. One example is the icosidodecahedron, shown in the drawings at the right.

1. What are the regular polygons that make up the faces of an icosidodecahedron? Describe the polygons that meet at each vertex. **Pentagons and triangles; at each vertex two regular pentagons and two equilateral triangles meet.**

Some of the Archimedean solids can be made by cutting off the corners of a regular polyhedron, or Platonic solid. Follow the steps below to make a truncated cube.

   a. Draw a cube (or make a cube out of modeling clay).

   b. Visualize a tilted plane that intersects the top, front, and right faces of the cube such that the cross section is an equilateral triangle. This plane should be closer to the corner of the cube than the midpoint of the edges. Draw this cross section (or cut off the corner of the cube of modeling clay).

   c. Repeat step (b) with the other corners of the cube (or the clay). All cross sections should be congruent.

2. What are the regular polygons that make up the faces of a truncated cube? Describe the polygons that meet at each vertex. **Octagons and triangles; at each vertex two octagons and a triangle meet.**

3. Suppose that the cross sections have been formed by planes that cut off the corners of the cube and intersect the midpoints of the edges. Draw or model the solid that would be formed. What are the regular polygons that make up the faces of this solid? **Squares and triangles**

4. Other Archimedean solids that can be made by cutting off the corners of a Platonic solid include the truncated tetrahedron and the truncated octahedron. Follow a set of steps similar to those above to create these solids. Then describe the faces of each. **Truncated tetrahedron: hexagonal and triangular faces; truncated octahedron: hexagonal and square faces**

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11-1 Reteaching
Space Figures and Cross Sections

A polyhedron is a three-dimensional figure with faces that are polygons. Faces intersect at edges, and edges meet at vertices.

Faces, vertices, and edges are related by Euler’s Formula: \( F + V = E + 2 \).

For two dimensions, such as a representation of a polyhedron by a net, Euler’s Formula is \( F + V = E + 1 \). (\( F \) is the number of regions formed by \( V \) vertices linked by \( E \) segments.)

Problem

What does a net for the doorstop at the right look like? Label the net with its appropriate dimensions.

Exercises

Complete the following to verify Euler’s Formula.

1. On graph paper, draw three other nets for the polyhedron shown above. Let each unit of length represent \( \frac{1}{2} \) in. Sample:

2. Cut out each net, and use tape to form the solid figure. Check students’ work.

3. Count the number of vertices, faces, and edges of one of the figures. 6 vertices, 5 faces, 9 edges

4. Verify that Euler’s Formula, \( F + V = E + 2 \), is true for this polyhedron. \( F + V = E + 2, 5 + 6 = 9 + 2, 11 = 11 \)

Draw a net for each three-dimensional figure. Samples:

5.

6.
A cross section is the intersection of a solid and a plane. Cross sections can be many different shapes, including polygons and circles.

The cross section of this solid and this plane is a rectangle. This cross section is a horizontal plane.

To draw a cross section, visualize a plane intersecting one face at a time in parallel segments. Draw the parallel segments, then join their endpoints and shade the cross section.

Exercises

Draw and describe the cross section formed by intersecting the rectangular prism with the plane described.

7. a plane that contains the vertical line of symmetry
   Answers may vary. a rectangle or a square

8. a plane that contains the horizontal line of symmetry
   a rectangle

9. a plane that passes through the midpoint of the top left edge, the midpoint of the top front edge, and the midpoint of the left front edge
   an isosceles triangle

10. What is the cross section formed by a plane that contains a vertical line of symmetry for the figure at the right?
    Answers may vary. a hexagon or a rectangle

11. Visualization  What is the cross section formed by a plane that is tilted and intersects the front, bottom, and right faces of a cube?
    a triangle
Complete the vocabulary chart by filling in the missing information.

<table>
<thead>
<tr>
<th>Word or Word Phrase</th>
<th>Definition</th>
<th>Picture or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>altitude of a prism or cylinder</td>
<td>A perpendicular segment that joins the planes of the bases of a prism or cylinder is the <em>altitude of a prism or cylinder</em>.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>bases of a prism or cylinder</td>
<td>1. The congruent, parallel faces on a prism or a cylinder are the <em>bases of a prism or cylinder</em>.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>height of a prism or cylinder</td>
<td>The <em>height of a prism or cylinder</em> is the length of an altitude of the solid. It is the distance between the solid’s bases.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>lateral area of a prism or cylinder</td>
<td>3. The <em>lateral area of a prism or cylinder</em> is the sum of the areas of the lateral (side) faces of a prism or cylinder.</td>
<td><img src="image" alt="Formula" /></td>
</tr>
<tr>
<td>surface area of a prism or cylinder</td>
<td>The <em>surface area of a prism or cylinder</em> is the sum of the lateral area and the areas of the two bases.</td>
<td><img src="image" alt="Formula" /></td>
</tr>
<tr>
<td>oblique prism or oblique cylinder</td>
<td>An <em>oblique prism or oblique cylinder</em> is a prism or cylinder with lateral faces and bases that are not perpendicular.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>right prism or right cylinder</td>
<td>6. A <em>right prism or right cylinder</em> is a prism or cylinder with lateral faces and bases that are perpendicular.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>
11-2 Think About a Plan
Surface Areas of Prisms and Cylinders

Reasoning  Suppose you double the radius of a right cylinder.
   a. How does that affect the lateral area?
   b. How does that affect the surface area?
   c. Use the formula for surface area of a right cylinder to explain why the
      surface area in part (b) was not doubled.

Understanding the Problem
1. What is the formula for the lateral area of a right cylinder? \( L.A. = 2\pi rh \)
2. What is the formula for the surface area of a right cylinder? \( S.A. = 2\pi rh + 2\pi r^2 \)

Planning the Solution
3. How does doubling the radius affect the formulas for the lateral and
   surface areas? In the formula for the surface area, where do you need
   to be most careful?
   Replace \( r \) with \( 2r \) everywhere it appears in each formula; in the formula for
   surface area, be careful to apply the exponent of 2 to \( 2r \), not just \( r \).
4. How do you compare the new formulas you get after doubling the radius in
   the original formulas?
   Factor each formula or divide the new formula by the old formula.

Getting an Answer
5. Write the formula for the new lateral area after the radius has been doubled.
   Compare this to the original formula for the lateral area. What effect does
   doubling the radius have?
   Original formula: \( L.A. = 2\pi rh \); the formula after doubling the radius is
   \( \text{New L.A.} = 2\pi(2r)h = 4\pi rh \); because \( \frac{\text{New L.A.}}{L.A.} = \frac{4\pi rh}{2\pi rh} = 2 \), doubling the radius
   doubles the lateral area.
6. Write the formula for the new surface area after the radius has been doubled.
   Compare this to the original formula for the surface area. What effect does
   doubling the radius have?
   Original formula: \( S.A. = 2\pi rh + 2\pi r^2 \); the formula after doubling the radius is
   \( \text{New S.A.} = 2\pi(2r)h + 2\pi(2r)^2 = 4\pi rh + 8\pi r^2 \); the area of the base has been
   quadrupled. So, the surface area has more than doubled.
11-2 Practice

Surface Areas of Prisms and Cylinders

Use a net to find the surface area of each prism. Round your answer to the nearest whole number.

1. \[ 192 \text{ ft}^2 \]

2. \[ 111 \text{ in.}^2 \]

3. a. Classify the prism at the right. \textit{pentagonal prism}

   b. The bases are regular pentagons. Find the lateral area of the prism. \[ 275 \text{ cm}^2 \]

   c. The area of each is \[ 43 \text{ cm}^2 \]. Find the sum of their areas. \[ 86 \text{ cm}^2 \]

   d. Find the surface area of the prism. \[ 361 \text{ cm}^2 \]

Use formulas to find the lateral area and surface area of each prism. Round your answer to the nearest whole number.

4. \[ 196 \text{ m}^2; 294 \text{ m}^2 \]

5. \[ 227; 260 \]

6. \[ 624 \text{ in.}^2; 681 \text{ in.}^2 \]

Find the lateral area of each cylinder to the nearest whole number.

7. \[ 126 \text{ m}^2 \]

8. \[ 346 \text{ m}^2 \]

9. \[ 968 \text{ m}^2 \]

10. A box of cereal measures 8 in. wide, 11 in. high, and 2 in. deep. If all surfaces are made of cardboard and the total amount of overlapping cardboard in the box is \[ 7 \text{ in.}^2 \], how much cardboard is used to make the cereal box? \[ 259 \text{ in.}^2 \]

11. Judging by appearances, what is the surface area of the solid shown at the right? Show your answer to the nearest whole number. \[ 1082 \text{ cm}^2 \]
Find the surface area of each cylinder in terms of $\pi$.

12. \[ d = 15 \text{ cm} \]
\[ 32 \text{ cm} \]
\[ 592.5\pi \text{ cm}^2 \]

13. \[ r = 10 \text{ in.} \]
\[ 21 \text{ in.} \]
\[ 620\pi \text{ in.}^2 \]

14. a. A cylindrical container of paint with radius 6 in. is 15 in. tall. If all of the surfaces except the top are made of metal, how much metal is used to make the container? Assume the thickness of the metal is negligible. Show your answer to the nearest square inch. \[ 679 \text{ in.}^2 \]

b. If the top of the paint container is made of plastic, how much plastic is used to make the top? Assume the thickness of the plastic is negligible. Show your answer to the nearest square inch. \[ 113 \text{ in.}^2 \]

15. a. \textbf{Reasoning} Suppose that a cylinder has a radius of $r$ units and a height of $2r$ units. The lateral area of the cylinder is $64\pi$ square units. What is the value of $r$? \[ 4 \text{ units} \]

b. What is the surface area of the cylinder? Round your answer to the nearest square unit. \[ 302 \text{ square units} \]

\textbf{Visualization} Suppose you revolve the plane region completely about the given line to sweep out a solid of revolution. Describe the solid and find its surface area in terms of $\pi$.

16. the $x$-axis \( \text{cylinder; } 56\pi \text{ square units} \)

17. the $y$-axis \( \text{cylinder; } 42\pi \text{ square units} \)

18. the line $x = 3$ \( \text{cylinder; } 42\pi \text{ square units} \)

19. the line $y = 2$ \( \text{cylinder; } 20\pi \text{ square units} \)

20. An artist creates a right prism whose bases are regular decagons. He wants to paint the surface of the prism. One can of paint can cover 32 square feet. How many cans of paint must he buy if the height of the prism is 11 ft and the length of each side of the decagon is 2.4 ft? The area of a base is approximately 89 ft$^2$. \[ 14 \text{ cans} \]

21. \textbf{Open-Ended} Draw a cylinder with a surface area of $136\pi \text{ cm}^2$.
\textbf{Check students’ drawings. Sample:} radius of bases = 2 cm and height = 32 cm, or radius of bases = 4 cm and height = 13 cm
Use a net to find the surface area of each prism.

1. 160 m\(^2\)

2. 84 ft\(^2\)

3. a. Classify the prism at the right. **right hexagonal prism**
   
b. Find the lateral area of the prism. 216 cm\(^2\)
   
c. The bases are regular hexagons. The area of each is about 41.6 cm\(^2\). Find the sum of their areas. 83.2 cm\(^2\)
   
d. Find the surface area of the prism. 299.2 cm\(^2\)

Use formulas to find the surface area of each prism. Round your answer to the nearest whole number.

4. To start, use the formula for the lateral area of a prism, then find the perimeter of the base trapezoid.
   \[ L.A. = ph \]
   \[ p = 3 + 5 + 6 + 7 = 21 \text{ m} \]

5. 150 m\(^2\)

6. 48 ft\(^2\)

7. A box measures 10 in. wide, 12 in. high, and 14 in. deep. If all surfaces are made of cardboard, how much cardboard is used to make the box? **856 \text{ in.}^2**

8. An artist creates a right prism whose bases are regular pentagons. He wants to paint the lateral surfaces of the prism. One can of paint can cover 30 ft\(^2\).
   How many cans of paint must he buy if the height of the prism is 15 ft and the length of each side of the pentagon is 5 ft? **13 cans**
11-2 Practice (continued) Form K

Surface Areas of Prisms and Cylinders

Find the surface area of each cylinder in terms of \( \pi \).

9. 

\[
\text{To start, use the formula for the surface area of the cylinder, then identify the variables and any given values.}
\]

\[
\text{S.A.} = 2\pi rh + 2\pi r^2
\]

\[
r = 4 \text{ cm, } h = 12 \text{ cm}
\]

10. 

\[
128\pi \text{ cm}^2
\]

11. 

\[
522\pi \text{ mm}^2
\]

Find the lateral area of each cylinder to the nearest whole number.

12. 

\[
113 \text{ in.}^2
\]

13. 

\[
d = 12 \text{ m}
\]

14. 

\[
754 \text{ m}^2
\]

15. Reasoning A cylinder has a height that is 2 times as large as its radius. The lateral area of the cylinder is \( 16\pi \) square units.

a. What is the length of the radius of the cylinder? 2 units

b. What is the height of the cylinder? 4 units

c. What is the surface area of the cylinder? Round your answer to the nearest square unit. 75 square units

16. Reasoning A triangular prism and a rectangular prism both have bases that are regular polygons with sides 2 units long. Which has a greater surface area? Explain.

The surface area of a rectangular prism is greater. It has a greater perimeter, and its bases have greater areas.
11-2 **Standardized Test Prep**

**Surface Areas of Prisms and Cylinders**

### Multiple Choice

For Exercises 1–8, choose the correct letter.

1. What is the lateral surface area of a cube with side length 9 cm?  
   - **B**  
   \[ \text{A} \ 72 \text{ cm}^2 \]  
   \[ \text{B} \ 324 \text{ cm}^2 \]  
   \[ \text{C} \ 405 \text{ cm}^2 \]  
   \[ \text{D} \ 486 \text{ cm}^2 \]

2. What is the surface area of a prism whose bases each have area 16 m\(^2\) and whose lateral surface area is 64 m\(^2\)?  
   - **G**  
   \[ \text{F} \ 80 \text{ m}^2 \]  
   \[ \text{G} \ 96 \text{ m}^2 \]  
   \[ \text{H} \ 144 \text{ m}^2 \]  
   \[ \text{I} \ 160 \text{ m}^2 \]

3. A cylindrical container with radius 12 cm and height 7 cm is covered in paper. What is the area of the paper? Round to the nearest whole number.  
   - **D**  
   \[ \text{A} \ 528 \text{ cm}^2 \]  
   \[ \text{B} \ 835 \text{ cm}^2 \]  
   \[ \text{C} \ 1055 \text{ cm}^2 \]  
   \[ \text{D} \ 1432 \text{ cm}^2 \]

For Exercises 4 and 5, use the prism at the right.

4. What is the surface area of the prism?  
   - **H**  
   \[ \text{F} \ 283.8 \text{ m}^2 \]  
   \[ \text{H} \ 325.4 \text{ m}^2 \]  
   \[ \text{C} \ 292.4 \text{ m}^2 \]  
   \[ \text{I} \ 407 \text{ m}^2 \]

5. What is the lateral surface area of the prism?  
   - **B**  
   \[ \text{A} \ 283.8 \text{ m}^2 \]  
   \[ \text{B} \ 292.4 \text{ m}^2 \]  
   \[ \text{C} \ 325.4 \text{ m}^2 \]  
   \[ \text{D} \ 407 \text{ m}^2 \]

For Exercises 6 and 7, use the cylinder at the right.

6. What is the lateral surface area of the cylinder?  
   - **H**  
   \[ \text{F} \ 12\pi \text{ cm}^2 \]  
   \[ \text{H} \ 216\pi \text{ cm}^2 \]  
   \[ \text{C} \ 18\pi \text{ cm}^2 \]  
   \[ \text{I} \ 288\pi \text{ cm}^2 \]

7. What is the surface area of the cylinder?  
   - **D**  
   \[ \text{A} \ 12\pi \text{ cm}^2 \]  
   \[ \text{B} \ 18\pi \text{ cm}^2 \]  
   \[ \text{C} \ 216\pi \text{ cm}^2 \]  
   \[ \text{D} \ 288\pi \text{ cm}^2 \]

8. The height of a cylinder is three times the diameter of the base. The surface area of the cylinder is 126\(\pi\) ft\(^2\). What is the radius of the base?  
   - **F**  
   \[ \text{F} \ 3 \text{ ft} \]  
   \[ \text{G} \ 6 \text{ ft} \]  
   \[ \text{H} \ 9 \text{ ft} \]  
   \[ \text{I} \ 18 \text{ ft} \]

### Short Response

9. What are the lateral area and the surface area of the prism?  
   - **[2] L.A. = 1200 \text{ in.}^2; S.A. = 1392 \text{ in.}^2**  
   - **[1] one of two answers correct [0] no correct response given**
11-2 Enrichment
Surface Areas of Prisms and Cylinders

Constructing Rectangular Boxes

Given a rectangular sheet of material, such as paper or metal, it is possible to cut out squares and rectangles and reassemble the result into a right rectangular prism. For example, the sheet of paper pictured in the diagram is 8 in. by 12 in.

Use the figure at the right for Exercises 1–15.

1. What is the area of the rectangle? \(96 \text{ in.}^2\)

2. Suppose that two 2-in. squares are cut out as indicated along the dotted lines. What is the total area of the two squares? \(8 \text{ in.}^2\)

3. These squares are to be used as bases of a right rectangular prism, and the remaining material is to be used to construct the lateral faces. After the bases have been cut out from the sheet, how much area remains? \(88 \text{ in.}^2\)

4. How many lateral faces will the rectangular prism have? 4

5. What must be the area of each face? \(22 \text{ in.}^2\)

6. What must be the height of the rectangular prism? 11 in.

7. What is the surface area of the rectangular prism? \(96 \text{ in.}^2\)

Now, suppose that each side of the square base is \(s\) in.

8. What is the total area of the bases of the rectangular prism that will be constructed? \((2s^2) \text{ in.}^2\)

9. How much area remains? \((96 - 2s^2) \text{ in.}^2\)

10. How many lateral faces will the rectangular prism have? 4

11. What must be the area of each face? \((24 - 0.5s^2) \text{ in.}^2\)

12. One length of each face is known because it is also the side of either the top or bottom. What is its length? \(s\) in.

13. What must be the height of the rectangular prism? \((\frac{24}{3} - 0.5s) \text{ in.}\)

14. What is the surface area of the rectangular prism? \(96 \text{ in.}^2\)

15. Predict the surface area of the rectangular prism created if two 4-in. squares are cut out of the rectangle above and used as the bases. Explain. \(96 \text{ in.}^2\); no matter what the size of the base is, the surface area of the rectangular prism will always be \(96 \text{ in.}^2\) as long as the entire piece of paper is used.
11-2 Reteaching
Surface Areas of Cylinders and Prisms

A prism is a polyhedron with two congruent parallel faces called bases. The non-base faces of a prism are lateral faces. The dimensions of a right prism can be used to calculate its lateral area and surface area.

The lateral area of a right prism is the product of the perimeter of the base and the height of the prism.

\[ \text{L.A.} = ph \]

The surface area of a prism is the sum of the lateral area and the areas of the bases of the prism.

\[ \text{S.A.} = \text{L.A.} + 2B \]

Problem

What is the lateral area of the regular hexagonal prism?

\[ \text{L.A.} = ph \]
\[ p = 6(4 \text{ in.}) = 24 \text{ in.} \quad \text{Calculate the perimeter.} \]
\[ \text{L.A.} = 24 \text{ in.} \times 13 \text{ in.} \quad \text{Substitute.} \]
\[ \text{L.A.} = 312 \text{ in.}^2 \quad \text{Multiply.} \]

The lateral area is 312 in.².

Problem

What is the surface area of the prism?

\[ \text{S.A.} = \text{L.A.} + 2B \]
\[ p = 2(7 \text{ m} + 8 \text{ m}) \quad \text{Calculate the perimeter.} \]
\[ p = 30 \text{ m} \quad \text{Simplify.} \]
\[ \text{L.A.} = ph \]
\[ \text{L.A.} = 30 \text{ m} \times 30 \text{ m} \quad \text{Substitute.} \]
\[ \text{L.A.} = 900 \text{ m}^2 \quad \text{Multiply.} \]
\[ B = 8 \text{ m} \times 7 \text{ m} \quad \text{Find base area.} \]
\[ B = 56 \text{ m}^2 \quad \text{Multiply.} \]
\[ \text{S.A.} = \text{L.A.} + 2B \]
\[ \text{S.A.} = 900 \text{ m}^2 + 2 \times 56 \text{ m}^2 \quad \text{Substitute.} \]
\[ \text{S.A.} = 1012 \text{ m}^2 \quad \text{Simplify.} \]

The surface area of the prism is 1012 m².
11-2 Reteaching (continued)
Surface Areas of Cylinders and Prisms

A cylinder is like a prism, but with circular bases. For a right cylinder, the radius of the base and the height of the cylinder can be used to calculate its lateral area and surface area.

Lateral area is the product of the circumference of the base (2πr) and the height of the cylinder. Surface area is the sum of the lateral area and the areas of the bases (2πr²).

\[
\text{L.A.} = 2\pi rh \quad \text{or} \quad \pi dh \\
\text{S.A.} = 2\pi rh + 2\pi r^2
\]

Problem

The diagram at the right shows a right cylinder. What are the lateral area and surface area of the cylinder?

\[
\text{L.A.} = 2\pi rh \quad \text{or} \quad \pi dh \\
\text{L.A.} = 2\pi \times 4 \text{ in.} \times 9 \text{ in.} \quad \text{Substitute for r and h.} \\
\text{L.A.} = 72\pi \text{ in.}^2 \quad \text{Multiply.}
\]

The lateral area is 72\pi \text{ in.}^2.

\[
\text{S.A.} = 2\pi rh + 2\pi r^2 \\
\text{S.A.} = 2\pi \times 4 \text{ in.} \times 9 \text{ in.} + 2\pi \times (4 \text{ in.})^2 \quad \text{Substitute for r and h.} \\
\text{S.A.} = 72\pi \text{ in.}^2 + 32\pi \text{ in.}^2 \quad \text{Multiply.} \\
\text{S.A.} = 104\pi \text{ in.}^2 \quad \text{Add.}
\]

The surface area is 104\pi \text{ in.}^2.

Exercises

Find the lateral area and surface area of each figure. Round your answers to the nearest tenth, if necessary.

1. \[
\text{L.A.} = 565.5 \text{ cm}^2; \quad \text{S.A.} = 722.6 \text{ cm}^2
\]

2. \[
\text{L.A.} = 576 \text{ in.}^2; \quad \text{S.A.} = 864 \text{ in.}^2
\]

3. \[
\text{L.A.} = 138.3 \text{ m}^2; \quad \text{S.A.} = 153.3 \text{ m}^2
\]

4. A cylindrical carton of raisins with radius 4 cm is 25 cm tall. If all surfaces except the top are made of cardboard, how much cardboard is used to make the raisin carton? Round your answer to the nearest square centimeter.

\[
\text{679 cm}^2
\]
11-3  
**Additional Vocabulary Support**  
**Surface Areas of Pyramids and Cones**

Use the list below to complete the web.

<table>
<thead>
<tr>
<th>Base is a circle.</th>
<th>Base is a polygon.</th>
<th>L.A. = $\frac{1}{2} p \ell$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L.A. = $\frac{1}{2} \cdot 2\pi r \ell$</td>
<td>pyramid</td>
<td>S.A = L.A + B</td>
</tr>
</tbody>
</table>

- **pyramid**
- **cone**

Base is a polygon.  
L.A. = $\frac{1}{2} p \ell$  
S.A = L.A + B  
L.A. = $\frac{1}{2} \cdot 2\pi r \ell$

Lateral area and base area are used to find surface area.  
half the product of the slant height and perimeter/circumference of the base
11-3  Think About a Plan
Surface Areas of Pyramids and Cones

Find the lateral area of the cone to the nearest whole number.

Understanding the Problem

1. What is the formula for the lateral area of a cone? \( \text{L.A.} = \pi rl \)

2. How are the two variables in this formula defined?
   - The radius, \( r \), is the distance from the center of the circle to a point on the circle. The slant height, \( l \), is the distance from the vertex of the cone to a point on the circle.

3. What two pieces of information are given in the figure of the cone?
   - the diameter and the height

Planning the Solution

4. How can you use the given information to find the radius?
   - Divide the length of the diameter by 2.

5. How can you use the given information and the radius to find the slant height?
   - Substitute the height and the radius into the Pythagorean Theorem, \( l = \sqrt{r^2 + h^2} \).

Getting an Answer

6. What is the radius? \( r = \frac{1}{2} \cdot 4 \text{ m} = 2 \text{ m} \)

7. What is the slant height of the cone? \( l = \sqrt{2^2 + (4.5)^2} = \sqrt{24.25} \text{ m} \)

8. What is the lateral area of the cone? \( \text{L.A.} = \pi \cdot 2 \cdot \sqrt{24.25} = 2\sqrt{24.25}\pi \approx 31 \text{ m}^2 \)
Find the lateral area of each pyramid to the nearest whole number.

1. \[ \frac{1}{2} \times 10 \times 10 = 100 \text{ m}^2 \]

2. \[ \frac{1}{2} \times 6.9 \times 10 = 34.5 \text{ m}^2 \]

Find the surface area of each pyramid to the nearest whole number.

3. \[ \frac{1}{2} \times 12 \times 12 = 72 \text{ m}^2 \] \[ + \text{ base area} = 297 \text{ m}^2 \]

4. \[ \frac{1}{2} \times 2 \times 30 = 30 \text{ cm}^2 \] \[ + \text{ base area} = 124 \text{ cm}^2 \]

5. \[ \frac{1}{2} \times 10 \times 10 = 50 \text{ m}^2 \] \[ + \text{ base area} = 128 \text{ m}^2 \]

Find the lateral area of each cone to the nearest whole number.

6. \[ \frac{1}{2} \times 6 \times 4 = 12 \text{ cm}^2 \]

7. \[ \frac{1}{2} \times 10 \times 13 \approx 65 \text{ cm}^2 \]

Find the surface area of each cone in terms of \( \pi \).

8. \[ \frac{1}{2} \times 8 \times 12 = 48 \pi \text{ cm}^2 \]

9. \[ \frac{1}{2} \times 16 \times 12 = 96 \pi \text{ cm}^2 \]

10. \[ \frac{1}{2} \times 12 \times 12 = 72 \pi \text{ cm}^2 \]

11. The surface area of a cone is \(16.8\pi \text{ in}^2 \). The radius is 3 in. What is the slant height? \(2.6 \text{ in} \)

12. The lateral area of a cone is \(155.25\pi \text{ m}^2 \). The slant height is 13.5 m. What is the radius? \(11.5 \text{ m} \)

13. The roof of a clock tower is a pentagonal pyramid. Each side of the base is 7 ft long. The slant height is 9 ft. What is the area of the roof? \(157.5 \text{ ft}^2 \)

14. Write a formula to show the relationship between lateral area and the length of a side of the base \(s\) and slant height in a square pyramid. \[ \text{L.A.} = \frac{1}{2}p\ell = \frac{1}{2}(4s)\ell = 2s\ell \]
Find the surface area to the nearest whole number.

15. \[
\begin{align*}
\text{6 cm} & \\
\text{10 cm} & \\
\text{14 cm} & \\
\text{10 cm} & \\
\end{align*}
\]
\[
553 \text{ cm}^2
\]

16. \[
\begin{align*}
\text{26 in.} & \\
\text{10 in.} & \\
\text{40 in.} & \\
\end{align*}
\]
\[
3644 \text{ in.}^2
\]

17. Write a formula to show the relationship between surface area and the length of a side of the base (s) and slant height in a square pyramid. \( S.A. = 2sl + s^2 \)

The length of a side (s) of the base, slant height (l), height (h), lateral area (L.A.), and surface area (S.A.) are measurements of a square pyramid. Given two of the measurements, find the other three to the nearest tenth.

18. \( s = 9 \text{ cm}, \ l = 14.5 \text{ cm} \) \( h = 13.8 \text{ cm}; \ L.A. = 261 \text{ cm}^2; \ S.A. = 342 \text{ cm}^2 \)

19. \( L.A. = 1542.4 \text{ m}^2, \ S.A. = 2566.4 \text{ m}^2 \) \( h = 18 \text{ m}; \ s = 32 \text{ m}; \ l = 24.1 \text{ m} \)

20. \( h = 9 \text{ cm}, \ l = 11.4 \text{ cm} \) \( s = 14.0 \text{ cm}; \ L.A. = 319.2 \text{ cm}^2; \ S.A. = 515.2 \text{ cm}^2 \)

Visualization Suppose you revolve the plane region completely about the given line to sweep out a solid of revolution. Describe the solid. Then find its surface area in terms of \( \pi \). Round to the nearest tenth.

21. about the \( y \)-axis \( \text{cone; } S.A. = 14.8\pi \text{ units}^2; 46.5 \text{ units}^2 \)

22. about the \( x \)-axis \( \text{cone; } S.A. = 51.9\pi \text{ units}^2; 163.0 \text{ units}^2 \)

23. about the line \( x = 2 \) \( \text{a cylinder with a cone cut out; } S.A. = 34.8\pi \text{ units}^2; 109.3 \text{ units}^2 \)

24. about the line \( y = 5 \) \( \text{a cylinder with a cone cut out; } S.A. = 71.9\pi \text{ units}^2; 225.9 \text{ units}^2 \)

25. Open-Ended Draw a cone with a lateral area of \( 28\pi \text{ cm}^2 \). Label its dimensions. Then find its surface area. \( \text{Check students’ drawings. Sample: radius of base = 4 cm and slant height = 7 cm; } 44\pi \text{ cm}^2 \)
Find the surface area of each pyramid to the nearest whole number.

1. To start, use the formula for surface area of the pyramid, then identify the variables and any given values. 
   \[ S.A. = \frac{1}{2}p\ell + B \]
   \[ p = 4 \cdot 7 = 28 \text{ in.} \]
   \[ \ell = 11 \text{ in.} \]
   \[ B = 7 \cdot 7 = 49 \text{ in}^2 \]
   \[ 203 \text{ in}^2 \]

2. \[ 132 \text{ m}^2 \]

3. \[ 145 \text{ m}^2 \]

Find the lateral area of each pyramid to the nearest whole number.

4. \[ 60 \text{ m}^2 \]

5. \[ 316 \text{ in}^2 \]

6. The figure at the right has one base and eight lateral faces. Find its surface area to the nearest whole number. \[ 320 \text{ m}^2 \]

7. The roof of a clock tower is a square pyramid. Each side of the base is 16 ft long. The slant height is 22 ft. What is the lateral area of the roof? \[ 704 \text{ ft}^2 \]

8. **Reasoning** Write a formula to show the relationship between surface area and the length of a side of the base (s) and slant height in a square pyramid. \[ S.A. = 2s\ell + s^2 \]

The length of a side (s) of the base, slant height (\( \ell \)), height (h), lateral area (L.A.), and surface area (S.A.) are measurements of a square pyramid. Given two of the measurements, find the other three to the nearest tenth.

9. \( s = 16 \text{ cm} \), \( \ell = 10 \text{ cm} \) \( h = 6 \text{ cm} \); L.A. = 320 cm\(^2\); S.A. = 576 cm\(^2\)

10. L.A. = 624 m\(^2\), S.A. = 1200 m\(^2\) \( s = 24 \text{ m} \); \( \ell = 13 \text{ m} \); \( h = 5 \text{ m} \)

11. \( h = 7 \text{ cm} \), \( \ell = 25 \text{ cm} \) \( s = 48 \text{ cm} \); L.A. = 2400 cm\(^2\); S.A. = 4704 cm\(^2\)
11-3 Practice (continued) Form K
Surface Areas of Pyramids and Cones

Find the surface area of each cone in terms of $\pi$.

12. To start, use the formula for surface area of the pyramid, then identify the variables and any given values.

$$S.A. = \pi r \ell + B$$

$$r = 4 \text{ mm}$$
$$\ell = 8 \text{ mm}$$

$$B = \pi \cdot \left(4^2\right) = 16\pi \text{ mm}^2$$

$$48\pi \text{ mm}^2$$

13. $$95\pi \text{ cm}^2$$

14. $$39\pi \text{ ft}^2$$

Find the lateral area of each cone to the nearest whole number.

15. $$3299 \text{ in.}^2$$

16. $$204 \text{ m}^2$$

17. Find the surface area of the figure at the right to the nearest whole number. (Hint: Add the base, the lateral area of the cylinder, and the lateral area of the cone.)

$$547 \text{ mm}^2$$

18. The lateral area of a cone is $60\pi \text{ m}^2$. The slant height is $15 \text{ m}$. What is the radius? $$4 \text{ m}$$

19. The surface area of a cone is $55\pi \text{ cm}^2$. The radius is $5 \text{ cm}$. What is the slant height? $$6 \text{ cm}$$
11-3 Standardized Test Prep  
Surface Areas of Pyramids and Cones

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. What is the lateral surface area of a square pyramid with side length 11.2 cm and slant height 20 cm?  
   A. 224 cm²  
   B. 448 cm²  
   C. 896 cm²  
   D. 2508.8 cm²

2. What is the lateral surface area of a cone with radius 19 cm and slant height 11 cm?  
   F. $19\pi$ cm²  
   G. $30\pi$ cm²  
   H. $200\pi$ cm²  
   I. $209\pi$ cm²

3. What is the lateral area of the square pyramid, to the nearest whole number?  
   A. 165 m²  
   B. 176 m²  
   C. 330 m²  
   D. 351 m²

4. What is the surface area of the cone, to the nearest whole number?  
   F. 221 cm²  
   G. 304 cm²  
   C. 240 cm²  
   I. 620 cm²

5. What is the surface area of a cone with diameter 28 cm and height 22 cm in terms of $\pi$?  
   A. $196\pi$ cm²  
   B. $365\pi$ cm²  
   C. $561.1\pi$ cm²  
   D. $2202.8\pi$ cm²

Extended Response

6. What are the perimeter of the base, slant height, lateral area, and surface area for the square pyramid, to the nearest tenth of a meter or square meter?

   [4] $p = 24$ m; $\ell = 12.4$ m; L.A. = $148.4$ m²; S.A. = $184.4$ m²
   [3] any three of the four values correctly given
   [2] any two of the four values correctly given
   [1] any one correct value given
   [0] no correct responses given
11-3 Enrichment
Surface Areas of Pyramids and Cones

Frustum of a Solid

A frustum of a pyramid or cone is the figure made when the tip of the pyramid or cone is cut off by a cross section that is perpendicular to the height. In a frustum of a pyramid or cone, there are two bases, an upper base and a lower base.

The figure at the upper right is a frustum of a square pyramid for which \( h \) is the height, \( a \) is the length of a side of the lower base, \( b \) is the length of a side of the upper base, and \( \ell \) is the slant height.

The figure at the lower right is a frustum of a cone for which \( h \) is the height, \( R \) is the radius of the lower base, \( r \) is the radius of the upper base, and \( \ell \) is the slant height.

Use the frustum of the regular square pyramid above to derive a surface area formula.

1. What is the area of its lower base? \( a^2 \)
2. What is the area of its upper base? \( b^2 \)
3. What is the shape of each lateral face? trapezoid
4. What is the area of each lateral face? \( \frac{1}{2}(a + b) \ell \)
5. What is the lateral area of the figure? \( 2\ell(a + b) \)
6. What is the formula for the surface area of a frustum of a square pyramid? \( a^2 + b^2 + 2\ell(a + b) \)

Use the frustum of the cone above to derive a surface area formula.

7. What is the area of its lower base? \( \pi R^2 \)
8. What is the area of its upper base? \( \pi r^2 \)
9. What is the lateral area of the figure? (Hint: Consider the formula for the lateral area of a cone.) \( \pi R\ell + \pi r\ell \)
10. What is the formula for the surface area of a frustum of a cone? \( \pi(R^2 + r^2 + R\ell + r\ell) \)

Find the surface area of each figure below. Round your answers to the nearest tenth.

11. \( 152 \text{ cm}^2 \)
12. \( 722.6 \text{ in}^2 \)
13. \( 506.6 \text{ ft}^2 \)
14. \( 1952 \text{ m}^2 \)
A pyramid is a polyhedron in which the base is any polygon and the lateral faces are triangles that meet at the vertex. In a regular pyramid, the base is a regular polygon. The height is the measure of the altitude of a pyramid, and the slant height is the measure of the altitude of a lateral face. The dimensions of a regular pyramid can be used to calculate its lateral area (L.A.) and surface area (S.A.).

\[ \text{L.A.} = \frac{1}{2} p \ell, \text{ where } p \text{ is the perimeter of the base and } \ell \text{ is slant height of the pyramid.} \]

\[ \text{S.A.} = \text{L.A.} + B, \text{ where } B \text{ is the area of the base.} \]

### Problem

What is the surface area of the square pyramid to the nearest tenth?

\[ \begin{align*}
\text{S.A.} &= \text{L.A.} + B \\
\text{L.A.} &= \frac{1}{2} p \ell \\
p &= 4(4 \text{ m}) = 16 \text{ m} \\
\ell &= \sqrt{2^2 + 10^2} = \sqrt{104} \\
\ell &= 10.2 \\
\text{L.A.} &= \frac{1}{2} (16\text{m})(10.2) = 81.6 \text{ m}^2 \\
B &= (4 \text{ m})(4 \text{ m}) = 16 \text{ m}^2 \\
\text{S.A.} &= 81.6 \text{ m}^2 + 16 \text{ m}^2 = 97.6 \text{ m}^2
\end{align*} \]

The surface area of the square pyramid is about 97.6 m².

### Exercises

Use graph paper, scissors, and tape to complete the following.

1. Draw a net of a square pyramid on graph paper. Sample:

2. Cut it out, and tape it together. Check students' work.

3. Measure its base length and slant height. Sample: base = 3 cm, slant height = 4 cm

4. Find the surface area of the pyramid. Sample: 33 cm²

In Exercises 5 and 6, round your answers to the nearest tenth, if necessary.

5. Find the surface area of a square pyramid with base length 16 cm and slant height 20 cm. 896 cm²

6. Find the surface area of a square pyramid with base length 10 in. and height 15 in. 416.2 in²
11-3 Reteaching (continued)

Surface Areas of Pyramids and Cones

A cone is like a pyramid, except that the base of a cone is a circle. The radius of the base and cylinder height can be used to calculate the lateral area and surface area of a right cone.

\[ \text{L.A.} = \pi r \ell, \text{ where } r \text{ is the radius of the base and } \ell \text{ is slant height of the cone.} \]

\[ \text{S.A.} = \text{L.A.} + B, \text{ where } B \text{ is the area of the base } (B = \pi r^2). \]

**Problem**

What is the surface area of a cone with slant height 18 cm and height 12 cm? Begin by drawing a sketch.

Use the Pythagorean Theorem to find \( r \), the radius of the base of the cone.

\[
\begin{align*}
r^2 + 12^2 &= 18^2 \\
r^2 + 144 &= 324 \\
r^2 &= 180 \\
r &\approx 13.4
\end{align*}
\]

Now substitute into the formula for the surface area of a cone.

\[
\begin{align*}
\text{S.A.} &= \text{L.A.} + B \\
&= \pi r \ell + \pi r^2 \\
&= \pi (13.4)(18) + 180\pi \\
&\approx 1323.2
\end{align*}
\]

The surface area of the cone is about 1323.2 cm\(^2\).

**In Exercises 7–10, round your answers to the nearest tenth, if necessary.**

7. Find the surface area of a cone with radius 5 m and slant height 15 m. \( 314.2 \text{ m}^2 \)

8. Find the surface area of a cone with radius 6 ft and height 11 ft. \( 349.3 \text{ ft}^2 \)

9. Find the surface area of a cone with radius 16 cm and slant height 20 cm. \( 1809.6 \text{ cm}^2 \)

10. Find the surface area of a cone with radius 10 in. and height 15 in. \( 880.5 \text{ in.}^2 \)
### Problem

What is the volume of the prism at the right? Justify and explain your work.

<table>
<thead>
<tr>
<th>Explain</th>
<th>Work</th>
<th>Justify</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First</strong>, write the formula for volume of a prism.</td>
<td>$V = Bh$</td>
<td>formula for volume of a prism</td>
</tr>
<tr>
<td><strong>Second</strong>, substitute the value of the height.</td>
<td>$V = B(2)$</td>
<td>substitution, $h = 2\text{ cm}$</td>
</tr>
</tbody>
</table>
| **Next**, substitute an expression for the area of the base. | $V = (6.2 \cdot 6)(2)$ | The base is a rectangle with side lengths $6\text{ cm}$ and $6.2\text{ cm}$.
| **Finally**, find the product. The volume of the rectangular prism is $74.4\text{ cm}^3$. | $V = 74.4$ | Multiply. |

**Solution**

$74.4\text{ cm}^3$

### Exercise

What is the volume of the prism at the right? Justify and explain your work.

<table>
<thead>
<tr>
<th>Explain</th>
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</tr>
<tr>
<td><strong>Second</strong>, substitute the value of the height.</td>
<td>$V = B(9)$</td>
<td>substitution, $h = 9\text{ cm}$</td>
</tr>
</tbody>
</table>
| **Next**, substitute an expression for the area of the base. | $V = \frac{1}{2} (3 \cdot 4.2)(9)$ | The base is a right triangle with legs $3\text{ cm}$ and $4.2\text{ cm}$.
| **Finally**, find the product. The volume of the rectangular prism is $56.7\text{ cm}^3$. | $V = 56.7$ | Multiply. |

**Solution**

$56.7\text{ cm}^3$
11-4 Think About a Plan
Volumes of Prisms and Cylinders

Swimming Pool  The approximate dimensions of an Olympic-size swimming pool are 164 ft by 82 ft by 6.6 ft.

a. Find the volume of the pool to the nearest cubic foot.
b. If 1 ft³ = 7.48 gallons, about how many gallons does the pool hold?

Understanding the Problem

1. What are the dimensions of the pool?
   164 ft by 82 ft by 6.6 ft

2. How long is the pool? How wide is the pool? How deep is the pool?
   164 ft long; 82 feet wide; 6.6 ft deep

3. What are the dimensions of the base of the pool?
   164 ft-by-82 ft

4. How do you convert cubic feet to gallons?
   Multiply the number of cubic feet by 7.48.

Planning the Solution

5. How would you use the dimensions of the pool to find its volume?
   First find the area of the base of the pool by substituting the values of the dimensions into the formula \( B = \ell \cdot w \), then find the volume by substituting the values for \( B \) and \( h \) into the formula \( V = B \cdot h \).

6. How would you use the volume of the pool to find the number of gallons the pool holds?
   Multiply the volume of the pool by 7.48.

Getting an Answer

7. What is the area of the base of the pool? \( B = \ell \cdot w = 164 \cdot 82 = 13,448 \text{ ft}^2 \)

8. What is the volume of the pool? \( V = B \cdot h = 13,448 \cdot 6.6 = 88,756.8 \text{ ft}^3 \)

9. About how many gallons of water does the pool hold?
   The pool holds about 88,756.8 \( \times 7.48 \approx 663,901 \) gallons.
Find the volume of each rectangular prism.

1. 1008 cm³
2. 225 cm³
3. 142.5 cm³
4. 456 m³
5. 82.6875 yd³
6. 56.35 in³

7. The base is a square, 4.5 cm on a side. The height is 5 cm. 101.25 cm³
8. The base is a rectangle with length 3.2 cm and width 4 cm. The height is 10 cm. 128 cm³

Find the volume of each triangular prism to the nearest tenth.

9. 2205 mm³
10. 3195.4 cm³
11. 1925 m³

12. The base is a right triangle with a leg of 12 in. and hypotenuse of 15 in. The height of the prism is 10 in. 540 in³
13. The base is a 30°-60°-90° triangle with a hypotenuse of 10 m. The height of the prism is 15 m. Find the volume to the nearest tenth. 324.8 m³

Find the volume of each cylinder in terms of π and to the nearest tenth.

14. 505.8π m³; 1588.9 m³
15. 172.1π cm³; 540.7 cm³
16. 40π mm³; 125.7 mm³

17. a right cylinder with a radius of 3.2 cm and a height of 10.5 cm 107.5π cm³; 337.8 cm³
18. a right cylinder with a diameter of 8 ft and a height of 15 ft. 240π ft²; 754 ft³
Find the volume of each composite figure to the nearest whole number.

19. \[ \text{589 ft}^3 \]

20. \[ \text{1872 cm}^3 \]

21. \[ \text{1214 in.}^3 \]

Find the value of \( x \) to the nearest tenth.

22. Volume: \( 576 \text{ cm}^3 \)

23. Volume: \( 980 \text{ mm}^3 \)

24. Volume: \( 602.88 \text{ cm}^3 \)

25. A cylindrical weather satellite has a diameter of 6 ft and a height of 10 ft. What is the volume available for carrying instruments and computer equipment, to the nearest tenth of a cubic foot? \( 282.7 \text{ ft}^3 \)

26. A No. 10 can has a diameter of 15.5 cm and a height of 17.5 cm. A No. 2.5 can has a diameter of 9.8 cm and a height of 11 cm. What is the difference in volume of the two can types, to the nearest cubic centimeter? \( 2472 \text{ cm}^3 \)

27. The NCAA recommends that a competition diving pool intended for use with two 1-m springboards and two 3-m springboards, in addition to diving platforms set at 5 m, 7.5 m, and 10 m above the water, have a width of 75 ft 1 in., a length of 60 ft, and a minimum water depth of 14 ft 10 in. What is the minimum volume of water such a pool would hold in cubic yards, to the nearest whole number? \( 2475 \text{ yd}^3 \)

28. What is the volume of the solid figure formed by the net? \( 320 \text{ m}^3 \)
Find the volume of each rectangular prism.

1. $160 \text{ cm}^3$
   - Base: $8 \text{ cm} 	imes 4 \text{ cm} 	imes 5 \text{ cm}$

2. $315 \text{ in.}^3$
   - Base: $9 \text{ in.} 	imes 5 \text{ in.} 	imes 7 \text{ in.}$

3. $630 \text{ m}^3$
   - Base: $15 \text{ m} 	imes 6 \text{ m} 	imes 7 \text{ m}$

4. $216 \text{ yd}^3$
   - Base: $6 \text{ yd} 	imes 6 \text{ yd} 	imes 6 \text{ yd}$

5. The base is a square, 9.6 cm on a side. The height is 6.2 cm. $571.392 \text{ cm}^3$

6. The base is a rectangle with length 4.7 cm and width 7.5 cm. The height is 6.1 cm. $215.025 \text{ cm}^3$

Find the volume of each triangular prism to the nearest tenth.

To start, use the formula for the volume of a triangular prism and the formula for the base area of a triangle.

$V = BH, B = \frac{1}{2}bh$

7. $120 \text{ m}^3$
   - Base: $6 \text{ m} 	imes 4 \text{ m} 	imes 10 \text{ m}$

8. $1385.6 \text{ ft}^3$
   - Base: $20 \text{ ft} 	imes 16 \text{ ft} 	imes 10 \text{ ft}$

9. The base is a right triangle with a leg of 8 in. and hypotenuse of 10 in. The height of the prism is 15 in. (Hint: Use the Pythagorean Theorem to find the length of the other leg.) $360 \text{ in.}^3$

10. The base is a $30^\circ$-$60^\circ$-$90^\circ$ triangle with a hypotenuse of 14 m. The height of the prism is 11 m. Find the volume to the nearest tenth. $466.8 \text{ m}^3$
Find the volume of each cylinder in terms of $\pi$ and to the nearest tenth.

To start, use the formula for the volume of a cylinder, then identify the variables and any given values.

$V = \pi r^2 h$

11. $24\pi \text{ m}^3; 75.4 \text{ m}^3$

12. $5880\pi \text{ cm}^3; 18,472.6 \text{ cm}^3$

13. The radius of the right cylinder is 6.3 cm. The height is 14.5 cm.

14. The diameter of the right cylinder is 16 ft. The height is 7 ft. $448\pi \text{ ft}^3; 1407.4 \text{ ft}^3$

Find the volume of each composite figure to the nearest whole number.

15. $105 \text{ m}^3$

16. $630 \text{ in.}^3$

Find the volume of each figure to the nearest tenth.

17. $560 \text{ mm}^3$

18. $169.6 \text{ ft}^3$

19. A cylindrical weather satellite has a diameter of 10 ft and a height of 6 ft. What is the volume available for carrying instruments and computer equipment, to the nearest tenth of a cubic foot? $471.2 \text{ ft}^3$

20. Can A has a diameter of 6 cm and a height of 6.5 cm. Can B has a diameter of 16 cm and a height of 11.5 cm. What is the difference in volume of the two can types, to the nearest cubic centimeter? $2128 \text{ cm}^3$
Gridded Response

Solve each exercise and enter your answer on the grid provided.

1. What is the volume in cubic inches of the prism?

2. What is the volume in cubic feet of the prism, rounded to the nearest cubic foot?

3. What is the volume in cubic inches of the cylinder, rounded to the nearest cubic inch?

4. What is \( x \), if the volume of the cylinder is \( 768\pi \text{ cm}^3 \)?

5. What is the volume in cubic inches of the solid figure, rounded to the nearest cubic inch?

Answers

1. 12.0
2. 25.0
3. 17.0
4. 48.0
5. 111.3
11-4 Enrichment
Volumes of Prisms and Cylinders

To describe positions in space, you need a three-dimensional coordinate system. This system is set up using three perpendicular lines, called the x-axis, y-axis, and z-axis. Points in a three-dimensional coordinate system are called ordered triples, or \((x, y, z)\). Point \(A\) is located at \((3, -2, 5)\).

Find the volume of each prism graphed on a three-dimensional coordinate system.

1. \(48 \text{ units}^3\)
2. \(30 \text{ units}^3\)
3. \(64 \text{ units}^3\)
4. \(15 \text{ units}^3\)

Use the given ordered triples to graph each prism on a three-dimensional coordinate system. Then find the volume of each.

5. right rectangular prism: \((3, 0, 0), (3, 5, 0), (0, 5, 0), (0, 0, 0), (3, 0, 6), (3, 5, 6), (0, 5, 6), (0, 0, 6)\)
   \(90 \text{ units}^3\)

6. right triangular prism: \((5, 0, 0), (0, -5, 0), (0, 0, 0), (5, 0, 2), (0, -5, 2), (0, 0, 2)\)
   \(25 \text{ units}^3\)
11-4 Reteaching
Volumes of Prisms and Cylinders

Problem

Which is greater: the volume of the cylinder or the volume of the prism?

Volume of the cylinder: \[ V = Bh \]
\[ = \pi r^2 \cdot h \]
\[ = \pi (3)^2 \cdot 12 \]
\[ \approx 339.3 \text{ in.}^3 \]

Volume of the prism: \[ V = Bh \]
\[ = s^2 \cdot h \]
\[ = 6^2 \cdot 12 \]
\[ = 432 \text{ in.}^3 \]

The volume of the prism is greater.

Exercises

Find the volume of each object.

1. the rectangular prism part of the milk container
   \[ 490 \text{ cm}^3 \]

2. the cylindrical part of the measuring cup
   \[ 77.175\pi \text{ or about } 242.5 \text{ cm}^3 \]

Find the volume of each of the following. Round your answers to the nearest tenth, if necessary.

3. a square prism with base length 7 m and height 15 m
   \[ 735 \text{ m}^3 \]

4. a cylinder with radius 9 in. and height 10 in.
   \[ 2544.7 \text{ in.}^3 \]

5. a triangular prism with height 14 ft and a right triangle base with legs measuring 9 ft and 12 ft
   \[ 756 \text{ ft}^3 \]

6. a cylinder with diameter 24 cm and height 5 cm
   \[ 2261.9 \text{ cm}^3 \]
11-4 Re-teaching (continued)
Volumes of Prisms and Cylinders

Problem

What is the volume of the triangular prism?

Sometimes the height of a triangular base in a triangular prism is not given. Use what you know about right triangles to find the missing value. Then calculate the volume as usual.

- hypotenuse = 18 cm
- short leg = 9 cm
- long leg = $9\sqrt{3}$ cm

Given

- $30^\circ$-$60^\circ$-$90^\circ$ triangle theorem
- $30^\circ$-$60^\circ$-$90^\circ$ triangle theorem

Volume of prism: $V = Bh$

$$V = \left(\frac{1}{2}\right)(9)(9\sqrt{3})(12)$$

$$V \approx 841.8 \text{ cm}^3$$

The volume of the triangular prism is about 841.8 cm$^3$.

Exercises

Find the volume of each prism. Round to the nearest tenth.

7. $22.5 \text{ cm}^3$

8. $2833.8 \text{ in.}^3$

9. $1208.0 \text{ mm}^3$

10. $7.1 \text{ in.}^3$

11. $42 \text{ m}^3$

12. $265.0 \text{ in.}^3$

Find the volume of each composite figure to the nearest tenth.

13. $76 \text{ ft}^3$

14. $96.8 \text{ in.}^3$

15. $111.4 \text{ in.}^3$
There are two sets of note cards below that show how to find the volume of a cone. The set on the left explains the thinking. The set on the right shows the work. Write the thinking and the steps in the correct order.

**Think Cards**
- Use a calculator.
- Substitute the values in the formula.
- Simplify.
- Round to the nearest cubic unit.
- Use the formula for volume of a cone.

**Write Cards**
- \[ V \approx 1072 \]
- \[ V = \frac{1}{3} \pi r^2 h \]
- \[ V = \frac{1}{3} \pi (8)^2 (16) \]
- \[ V = 341\frac{1}{3} \pi \]
- \[ V \approx 1072.330292 \]

**Think**
- First, use the formula for volume of a cone.
- Second, substitute the values in the formula.
- Next, simplify.
- Then, use a calculator.
- Finally, round to the nearest cubic unit.

**Write**
- Step 1 \[ V = \frac{1}{3} \pi r^2 h \]
- Step 2 \[ V = \frac{1}{3} \pi (8)^2 (16) \]
- Step 3 \[ V = 341\frac{1}{3} \pi \]
- Step 4 \[ V \approx 1072.330292 \]
- Step 5 \[ V \approx 1072 \]
Think About a Plan
Volumes of Pyramids and Cones

Writing  The two cylinders pictured at the right are congruent. How does the volume of the larger cone compare to the total volume of the two smaller cones? Explain.

Understanding the Problem

1. You are told that the two cylinders are congruent. What does that tell you about the cones?
   
   It tells you that the larger cone is twice as tall as the smaller cones.

2. You are asked to compare the volumes of the cones. Can you find the exact volumes of the cones? In what other way can you compare their volumes?
   
   No; you can compare their volumes using the formulas for the volumes of the cones.

Planning the Solution

3. What are the variables in the formula for the volume of a cone? What do they represent?
   
   The variables are the radius of the base, \( r \), and the height of the cone, \( h \).

4. Let \( r \) represent the radius of the larger cone. What is the radius of the smaller cones in terms of \( r \)? Let \( h \) represent the height of the larger cone. What is the height of the smaller cones in terms of \( h \)? Explain.
   
   Because the radii are equal, the radius of the smaller cones is also \( r \); because the height of the smaller cones is one-half the height of the larger cone, the height of the smaller cones is \( \frac{1}{2}h \).

Getting an Answer

5. What is the formula for the volume of the larger cone? What is the formula for the volume of the smaller cones?
   
   \[
   \text{larger cone: } V = \frac{1}{3}\pi r^2h; \text{ smaller cones: } V = \frac{1}{3}\pi r^2 \left(\frac{1}{2}h\right) \]

6. How does the volume of the larger cone compare to the total volume of the smaller cones?
   
   Because \( \frac{1}{3}\pi r^2 \left(\frac{1}{2}h\right) + \frac{1}{3}\pi r^2 \left(\frac{1}{2}h\right) = \frac{1}{3}\pi r^2h \), the volume of the larger cone is equal to the total volume of the two smaller cones.
Find the volume of each square pyramid. Round to the nearest tenth if necessary.

1. \[8 \text{ in.} \times 8 \text{ in.} \times 10 \text{ in.} = 213.3 \text{ in.}^3\]

2. \[48 \text{ cm}^3\]

3. \[15 \text{ cm}^3\]

Find the volume of each square pyramid, given its slant height. Round to the nearest tenth.

4. \[1.2 \text{ cm} \times 1.3 \text{ cm}^3\]

5. \[13 \text{ mm} \times 12.8 \text{ mm} = 621.2 \text{ mm}^3\]

6. \[5 \text{ m} \times 14.2 \text{ m} = 116.5 \text{ m}^3\]

7. The base of a pyramid is a square, 4.5 cm on a side. The height is 5 cm. Find the volume. \[33.75 \text{ cm}^3\]

8. The base of a pyramid is a square, 3.2 cm on a side. The height is 10 cm. Find the volume to the nearest tenth. \[34.1 \text{ cm}^3\]

Find the volume of each cone in terms of \(\pi\) and also rounded as indicated.

9. nearest cubic foot \[24 \text{ ft} \times 14 \text{ ft} \times 14 \text{ ft} = 392\pi \text{ ft}^3; 1232 \text{ ft}^3\]

10. nearest cubic meter \[12 \text{ m} \times 8 \text{ m} \times 8 \text{ m} = 64\pi \text{ m}^3; 201 \text{ m}^3\]

11. nearest cubic inch \[5 \text{ in.} \times 15 \text{ in.} \times 15 \text{ in.} = 31.25\pi \text{ in}^3; 98 \text{ in}^3\]

12. The base has a radius of 16 cm and a height of 12 cm. Round to the nearest cubic centimeter. \[1024\pi \text{ cm}^3; 3217 \text{ cm}^3\]

13. The base has a diameter of 24 m and a height of 15.3 m. Round to the nearest cubic meter. \[734.4\pi \text{ m}^3; 2307 \text{ m}^3\]

Find the volume to the nearest whole number.

14. \[5542 \text{ cm}^3\]

15. \[5089 \text{ cm}^3\]

16. \[5131 \text{ m}^3\]
Find the volume of each figure to the nearest whole number.

17.  
\[ \text{Volume} = 4835 \text{ cm}^3 \]

18.  
\[ \text{Volume} = 1433 \text{ m}^3 \]

19.  
\[ \text{Volume} = 938 \text{ mm}^3 \]

**Algebra** Find the value of \( x \) in each figure. Leave answers in simplest radical form. The diagrams are not to scale.

20.  
\[ \text{Volume} = 1500 \]

21.  
\[ \text{Volume} = 8\pi \]

22.  
\[ \text{Volume} = 126 \]

23. One right circular cone is set inside a larger right circular cone. The cones share the same axis, the same vertex, and the same height. Find the volume of the space between the cones if the diameter of the inside cone is 6 in., the diameter of the outside cone is 9 in., and the height of both is 5 in. Round to the nearest tenth. 58.9 in.\(^3\)

24. Some Native Americans still use tepees for special occasions and ceremonial purposes. Each group attending a family reunion, for example, might bring a small tepee, while using a larger tepee like the one pictured at the right for gathering together. The many poles form a rough cone with a circular base. What is the approximate volume of air in the tepee at the right, to the nearest cubic foot? 1672 ft\(^3\)

**Visualization** Suppose you revolve the plane region completely about the given line to sweep out a solid of revolution. Describe the solid. Then find its volume in terms of \( \pi \).

25. the \( x \)-axis  cone; \( 4\pi \)  
26. the \( y \)-axis  cone; \( 6\pi \)  
27. the line \( x = 3 \)  cylinder with cone cut out; \( 12\pi \)  
28. the line \( y = -2 \)  cylinder with cone cut out; \( 8\pi \)
11-5 Practice
Volumes of Pyramids and Cones

Find the volume of each square pyramid. Round to the nearest tenth if necessary.

To start, use the formula for the volume of a pyramid. Then find the area of the base of the pyramid.

\[ V = \frac{1}{3}Bh \]

1. \[ \text{10 cm} \]
   \[ \text{15 cm} \]
   \[ 750 \text{ cm}^3 \]

2. \[ \text{6 cm} \]
   \[ \text{7 cm} \]
   \[ 98 \text{ cm}^3 \]

Find the volume of each square pyramid, given its slant height. Round to the nearest whole number.

To start, find the height of the pyramid using the Pythagorean Theorem. Then use the formula for the volume of a pyramid.

3. \[ \text{32 cm} \]
   \[ 8662 \text{ cm}^3 \]

4. \[ \text{8 m} \]
   \[ 63 \text{ m}^3 \]

5. The base of a pyramid is a square, 24 cm on a side. The height is 13 cm. Find the volume. \[ 2496 \text{ cm}^3 \]

6. The base of a pyramid is a square, 14 cm on a side. The height of the pyramid is 25 cm. Find the volume to the nearest whole number. \[ 1633 \text{ cm}^3 \]

Find the volume of each cone in terms of \( \pi \) and also rounded as indicated.

7. nearest cubic foot
   \[ 4.2 \text{ ft} \]
   \[ 7 \text{ ft} \]
   \[ 41.16\pi \text{ ft}^3; 129 \text{ ft}^3 \]

8. nearest cubic inch
   \[ 42 \text{ in.} \]
   \[ 110 \text{ in.} \]
   \[ 16,170\pi \text{ in.}^3; 50,800 \text{ in.}^3 \]

9. The base has a radius of 8 cm and a height of 5 cm. Round to the nearest cubic centimeter. \[ 106\frac{2}{3} \text{ cm}^3; 335 \text{ cm}^3 \]

10. The base has a diameter of 20 m and a height of 12.6 m. Round to the nearest cubic meter. \[ 420\pi \text{ m}^3; 1319 \text{ m}^3 \]
Find the volume of each figure to the nearest whole number.

11. \[302 \text{ cm}^3\]

12. \[1257 \text{ m}^3\]

13. \[224 \text{ cm}^3\]

14. \[503 \text{ cm}^3\]

15. \[114 \text{ unit}^3\]

16. \[302 \text{ unit}^3\]

17. One right circular cone is set inside a larger right circular cone. Find the volume of the space between the cones if the diameter of the inside cone is 9 in., the diameter of the outside cone is 15 in., and the height of both is 8 in. Round to the nearest tenth. \[301.6 \text{ in.}^3\]

18. The Pyramid of Khufu is a square pyramid which had a side length of about 230 m and a height of about 147 m when it was completed. The Pyramid of Khafre had a side length of about 215 m and a height of about 144 m when it was completed. What was the approximate difference in the volume of the two pyramids upon completion? \[373,300 \text{ m}^3\]
11-5 Standardized Test Prep
Volumes of Pyramids and Cones

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. What is the volume of the pyramid? B
   \[ \text{A} \rightarrow 56 \text{ ft}^3 \] \[ \text{C} \rightarrow 196 \text{ ft}^3 \]
   \[ \text{D} \rightarrow 130 \frac{2}{3} \text{ ft}^3 \]

2. What is the volume of the cone, rounded to the nearest cubic inch? G
   \[ \text{F} \rightarrow 72 \text{ in.}^3 \] \[ \text{H} \rightarrow 905 \text{ in.}^3 \]
   \[ \text{I} \rightarrow 226 \text{ in.}^3 \] \[ \text{J} \rightarrow 2714 \text{ in.}^3 \]

3. What is the volume of the figure? B
   \[ \text{A} \rightarrow 15 \text{ cm}^3 \] \[ \text{C} \rightarrow 45 \text{ cm}^3 \]
   \[ \text{D} \rightarrow 33 \text{ cm}^3 \] \[ \text{E} \rightarrow 54 \text{ cm}^3 \]

4. What is the value of \( x \), if the volume of the cone is \( 12\pi \text{ m}^3 \)? F
   \[ \text{F} \rightarrow 4 \text{ m} \] \[ \text{H} \rightarrow 6 \text{ m} \]
   \[ \text{G} \rightarrow 5 \text{ m} \] \[ \text{I} \rightarrow 10 \text{ m} \]

5. What is the diameter of a cone with height 8 m and volume \( 150\pi \text{ m}^3 \)? D
   \[ \text{A} \rightarrow 7.5 \text{ m} \] \[ \text{B} \rightarrow 5\sqrt{3} \text{ m} \]
   \[ \text{C} \rightarrow 7.5\sqrt{3} \text{ m} \] \[ \text{D} \rightarrow 15 \text{ m} \]

Short Response

6. Error Analysis A student calculates the volume of the given cone as approximately 2094 cm\(^3\). Explain the error in the student’s reasoning and find the actual volume of the cone rounded to the nearest whole number.
   \( \text{[2]} \) The student uses slant height of the cone instead of the height to find the volume. The actual volume is approximately 1814 cm\(^3\). \( \text{[1]} \) incorrect explanation or incorrect volume \( \text{[0]} \) no correct responses given
It is possible to devise a formula for the volume of a regular pyramid that involves only the length of a side of the base and the slant height of the pyramid. Suppose that a regular pyramid has a square base whose side is $s$ and whose slant height is $\ell$.

1. Where does the vertex of the pyramid lie?  
   **on a line perpendicular to the base at the center of the square**

Assume that a plane perpendicular to the base is passed through the vertex of the pyramid so that the plane intersects the midpoints of two opposite sides of the square.

2. Draw a picture of the intersection of this plane with the pyramid.

3. What type of figure is formed? **isosceles triangle**

4. Draw the altitude of this triangle, and label it $h$.
   In terms of the pyramid, what is $h$? **height of the pyramid**

5. What theorem expresses $h$ in terms of $\ell$ and $s$? **Pythagorean Theorem**

6. Express $h$ in terms of $\ell$ and $s$.  
   $h = \frac{1}{2}\sqrt{4\ell^2 - s^2}$

7. What is the area of the base of the pyramid?  
   $A = s^2$

8. What is the volume of the pyramid?  
   $V = \frac{s^2}{6}\sqrt{4\ell^2 - s^2}$

Suppose that the base of the pyramid is a regular $n$-gon with a side of length $s$. 
Pass a plane through the vertex of the pyramid perpendicular to the base so that it intersects the midpoint of a side of the pyramid.

9. Draw the intersection. Let $V$ denote the vertex of the pyramid, $C$ the center of the base, and $M$ the midpoint of a side of the polygon.

10. What does $VM$ represent? **slant height**

11. What does $CM$ represent in terms of the polygon? **apothem**

12. What does $VC$ represent in terms of the pyramid? **height**

13. Let $VC = h$ and $CM = a$. Compute $h$ in terms of $a$ and $\ell$.  
   $h = \sqrt{\ell^2 - a^2}$

14. Find the area of the base in terms of $n$, $s$, and $a$.  
   $A = \frac{ns}{2}$

15. What is the volume of the pyramid?  
   $V = \frac{ns}{6}\sqrt{\ell^2 - a^2}$
Reteaching
Volumes of Pyramids and Cones

Problem
What is the volume of the square pyramid?

Sometimes the height of a triangular face in a square pyramid is not given. Here the slant height and the lengths of the sides of the base are given. Use what you know about right triangles to find the missing value. Then calculate the volume as usual.

\[ 7^2 + x^2 = 25^2 \]
\[ 49 + x^2 = 625 \]
\[ x^2 = 625 - 49 \]
\[ x^2 = 576 \]
\[ x = 24 \text{ cm} \]

Use the Pythagorean Theorem.
Substitute.
Isolate the variable.
Simplify.
Find the square root of each side.

Volume of the pyramid:

\[ V = \frac{1}{3} Bh \]
\[ = \frac{1}{3}(14 \times 14)(24) \]
\[ = 1568 \text{ cm}^3 \]

Use the formula for volume of a pyramid.
Substitute.
Simplify.

The volume of the square pyramid is 1568 cm\(^3\).

Exercises
Find the volume of each pyramid. Round to the nearest whole number.

1. \[ 54 \text{ cm} \]
\[ 45 \text{ cm} \]
\[ 34,992 \text{ cm}^3 \]

2. \[ 32 \text{ in.} \]
\[ 34 \text{ in.} \]
\[ 10,240 \text{ in.}^3 \]

3. \[ 150 \text{ m}^2 \]
\[ 3 \text{ m} \]
\[ 150 \text{ m}^3 \]

4. \[ 13 \text{ in.} \]
\[ 10 \text{ in.} \]
\[ 400 \text{ in.}^3 \]

5. \[ 36 \text{ yd} \]
\[ 400 \text{ yd}^2 \]
\[ 4800 \text{ yd}^3 \]

6. \[ 18 \text{ cm} \]
\[ 8 \text{ cm}^2 \]
\[ 48 \text{ cm}^3 \]
11-5  **Reteaching** (continued)

**Volumes of Pyramids and Cones**

**Problem**

What is the volume of the cone?

Find the height of the cone.

\[13^2 = h^2 + 5^2\]  
Use the Pythagorean Theorem.

\[169 = h^2 + 25\]  
Substitute.

\[h^2 = 144\]  
Simplify.

\[h = 12\]  
Take the square root of each side.

Find the volume of the cone.

\[V = \frac{1}{3}\pi r^2 h\]  
Use the formula for the volume of a cone.

\[= \frac{1}{3}\pi(5)^2 \cdot 12\]  
Substitute.

\[= 100\pi\]  
Simplify.

\[= 314.2\]  

The volume of the cone is about 314.2 cm\(^2\).

**Exercises**

7. From the figures shown below, choose the pyramid with volume closest to the volume of the cone at the right.

   **A.**
   
   - 7 in.
   - 5 in.
   - 5 in.
   - 3 in.

   **B.**
   
   - 15 in.
   - 3 in.
   - 3 in.

   **C.**
   
   - 7 in.
   - 8 in.
   - 8 in.
   - 2.5 in.

Find the volume of each figure. Round your answers to the nearest tenth.

8.  
   
   - 10 cm
   - 6 cm
   - \(301.6\) cm\(^3\)

9.  
   
   - 15 cm
   - 17 m
   - 1005.3 m\(^3\)

10.  
    
    - 2 ft
    - 6 ft
    - 18.8 ft\(^3\)

11.  
    
    - 12.7 m
    - 4.1 m
    - 211.6 m\(^3\)
For Exercises 1–6, match the term in Column A with its definition in Column B. The first one is done for you.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>sphere</td>
<td>a segment with both endpoints on a sphere that passes through the center of the sphere</td>
</tr>
<tr>
<td>1. center of a sphere</td>
<td>one of the two parts of a sphere on either side of a great circle</td>
</tr>
<tr>
<td>2. radius of a sphere</td>
<td>the set of all points in space that are a given distance from a given point</td>
</tr>
<tr>
<td>3. diameter of a sphere</td>
<td>a segment that has one endpoint at the center of a sphere and the other endpoint on the sphere</td>
</tr>
<tr>
<td>4. great circle</td>
<td>a point that is the same distance from all points on a sphere</td>
</tr>
<tr>
<td>5. circumference of a sphere</td>
<td>the circumference of any great circle of a sphere</td>
</tr>
<tr>
<td>6. hemisphere</td>
<td>the intersection of a sphere and the plane containing the center of a sphere</td>
</tr>
</tbody>
</table>

For Exercises 7–10, match the term in Column A with its formula in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. circumference of a sphere</td>
<td>$V = \frac{4}{3}\pi r^3$</td>
</tr>
<tr>
<td>8. surface area of a sphere</td>
<td>$C = 2\pi r$</td>
</tr>
<tr>
<td>9. volume of a sphere</td>
<td>$S.A. = 4\pi r^2$</td>
</tr>
<tr>
<td>10. diameter of a sphere</td>
<td>$D = 2r$</td>
</tr>
</tbody>
</table>
Think About a Plan
Surface Areas and Volumes of Spheres

Meteorology On September 3, 1970, a hailstone with diameter 5.6 in. fell at Coffeyville, Kansas. It weighed about 0.018 lb/in.\(^3\) compared to the normal 0.033 lb/in.\(^3\) for ice. About how heavy was this Kansas hailstone?

Understanding the Problem

1. What is the situation described in the problem?
   A hailstone fell from the sky.

2. What key piece of information is unstated or implicit?
   The hailstone is spherical.

3. What question do you have to answer?
   How heavy was the hailstone?

4. Is there any unnecessary information? Explain.
   Yes; we don’t need to know the normal lb/in.\(^3\) of ice.

Planning the Solution

5. What formula can you use to find the volume of the hailstone?
   You can use the formula \(V = \frac{4}{3} \pi r^3\).

6. How can you use the volume of the hailstone to find its weight?
   Multiply the volume by the density, 0.018 lb/in.\(^3\).

Getting an Answer

7. What is the volume of the hailstone? Show your work.
   \(V = \frac{4}{3} \pi (\frac{5.6}{2})^3 \approx 92 \text{ in.}^3\)

8. How do you find the weight of the hailstone?
   The weight is 92 \(\times\) 0.018 = 1.656 lb or 1 lb 10.5 oz.
11-6 Practice
Surface Areas and Volumes of Spheres

Find the surface area of the sphere with the given diameter or radius. Leave your answer in terms of \( \pi \).

1. \( d = 8 \text{ ft} \) \( 64\pi \text{ ft}^2 \)
2. \( r = 10 \text{ cm} \) \( 400\pi \text{ cm}^2 \)
3. \( d = 14 \text{ in.} \) \( 196\pi \text{ in.}^2 \)
4. \( r = 3 \text{ yd} \) \( 36\pi \text{ yd}^2 \)

Find the surface area of each sphere. Leave each answer in terms of \( \pi \).

5. \( \text{S.A.} = 100\pi \text{ cm}^2 \)
6. \( \text{S.A.} = 64\pi \text{ yd}^2 \)
7. \( \text{S.A.} = 36\pi \text{ ft}^2 \)
8. \( \text{S.A.} = 324\pi \text{ in.}^2 \)
9. \( \text{S.A.} = 400\pi \text{ mm}^2 \)
10. \( \text{S.A.} = 81\pi \text{ m}^2 \)

Use the given circumference to find the surface area of each spherical object. Round your answer to the nearest whole number.

11. an asteroid with \( C = 83.92 \text{ m} \) \( 2242 \text{ m}^2 \)
12. a meteorite with \( C = 26.062 \text{ yd} \) \( 216 \text{ yd}^2 \)
13. a rock with \( C = 16.328 \text{ ft} \) \( 85 \text{ ft}^2 \)
14. an orange with \( C = 50.24 \text{ mm} \) \( 803 \text{ mm}^2 \)

Find the volume of each sphere. Give each answer in terms of \( \pi \) and rounded to the nearest cubic unit.

15. \( \text{V} = 972\pi \text{ mm}^3; 3054 \text{ mm}^3 \)
16. \( \text{V} = 166\frac{2}{3}\pi \text{ yd}^3; 524 \text{ yd}^3 \)
17. \( \text{V} = 36\pi \text{ m}^3; 113 \text{ m}^3 \)
18. \( \text{V} = 1333\frac{1}{3}\pi \text{ in.}^3; 4189 \text{ in.}^3 \)
19. \( \text{S.A.} = 16\pi \text{ cm}^2 \)
20. \( \text{S.A.} = 64\pi \text{ cm}^2 \)

A sphere has the volume given. Find its surface area to the nearest whole number.

21. \( V = 1200 \text{ ft}^3 \) \( 546 \text{ ft}^2 \)
22. \( V = 750 \text{ m}^3 \) \( 399 \text{ m}^2 \)
23. \( V = 4500 \text{ cm}^3 \) \( 1318 \text{ cm}^2 \)
Find the volume in terms of \( \pi \) of each sphere with the given surface area.

24. \( 900\pi \text{ in.}^2 \quad 4500\pi \text{ in.}^3 \)  
25. \( 81\pi \text{ in.}^2 \quad 121.5\pi \text{ in.}^3 \)  
26. \( 6084\pi \text{ m}^2 \quad 79,092\pi \text{ m}^3 \)

27. The difference between drizzle and rain has to do with the size of the drops, not how much water is actually falling from the sky. If rain consists of drops larger than 0.02 in. in diameter, and drizzle consists of drops less than 0.02 in. in diameter, what can you say about the surface area and volume of rain and drizzle?

rain: S.A. > 0.0013 in.\(^2\), \( V > 0.000004 \text{ in.}^3 \); drizzle: S.A. < 0.0013 in.\(^2\), \( V < 0.000004 \text{ in.}^3 \)

28. A spherical scoop of ice cream with a diameter of 4 cm rests on top of a sugar cone that is 10 cm deep and has a diameter of 4 cm. If all of the ice cream melts into the cone, what percent of the cone will be filled? 80%

29. Point \( A \) is the center of the sphere. Point \( C \) is on the surface of the sphere. Point \( B \) is the center of the circle that lies in plane \( P \) and includes point \( C \). The radius of the circle is 12 mm. \( AB = 5 \text{ mm} \). What is the volume of the sphere to the nearest cubic mm? 9203 mm\(^3\)

30. **Writing** What are the formulas for the volumes of a sphere, a cone with a height equal to its radius, and a cylinder with its height equal to its radius? How are these formulas related?

Sphere: \( V = \frac{4}{3}\pi r^3 \); cone: \( V = \frac{1}{3}\pi r^3 \); cylinder: \( V = \pi r^3 \); the volume of a sphere is equal to the sum of the volume of a cone and a cylinder with height equal to their radii.

31. Candlepin bowling balls have no holes in them and are smaller than the bowling balls used in tenpin bowling. The regulation size is 4.5 in. in diameter, and their density is 0.05 lb/in.\(^3\). What is the regulation weight of a candlepin bowling ball? Round your answer to the nearest tenth of a pound. 2.4 lb

32. Find the radius of a sphere such that the ratio of the surface area in square inches to the volume in cubic inches is 4 : 1. 0.75 in.

33. Find the radius of a sphere such that the ratio of the surface area in square feet to the volume in cubic feet is 2 : 5. 7.5 ft or 7 ft 6 in.
Find the surface area of the sphere with the given diameter or radius. Leave your answer in terms of $\pi$.

1. $r = 6$ ft $144\pi$ ft$^2$
2. $d = 10$ cm $100\pi$ cm$^2$
3. $r = 8$ in. $256\pi$ in.$^2$
4. $d = 4$ yd $16\pi$ yd$^2$

Find the surface area of each sphere. Leave each answer in terms of $\pi$.

5. To start, use the formula for the surface area of a sphere. Then determine the radius and substitute it into the formula.

   \[ S.A. = 4\pi r^2 = 4\pi \cdot 3^2 \]

   $36\pi$ cm$^2$

6. $196\pi$ yd$^2$

7. $625\pi$ in.$^2$

8. $900\pi$ mm$^2$

Use the given circumference to find the surface area of each spherical object. Round your answer to the nearest tenth.

9. a baseball with $C = 9.25$ in. 27.2 in.$^2$
10. a softball with $C = 28.25$ cm 254.0 cm$^2$
11. a basketball with $C = 2.98$ ft 2.8 ft$^2$
12. a bowling ball with $C = 26.7$ in. 226.9 in.$^2$

Find the volume of each sphere. Give each answer in terms of $\pi$ and rounded to the nearest cubic unit.

13. To start, use the formula for the surface area of a sphere. Then determine the radius, and substitute it into the formula.

   \[ V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 6^3 \]

   $288\pi$ mm$^3$; 905 mm$^3$

14. $972\pi$ m$^3$; 3054 m$^3$

15. $562\frac{1}{2}\pi$ in.$^3$; 1767 in.$^3$

16. $7776\pi$ cm$^3$; 24,429 cm$^3$
A sphere has the volume given. Find its surface area to the nearest whole number.

17. \( V = 14,130 \text{ ft}^3 \quad 2826 \text{ ft}^2 \)

18. \( V = 4443 \text{ m}^3 \quad 1307 \text{ m}^2 \)

19. \( V = 100 \text{ in.}^3 \quad 104 \text{ in.}^2 \)

20. \( V = 31,400 \text{ mi}^3 \quad 4813 \text{ mi}^2 \)

21. A spherical scoop of ice cream with a diameter of 5 cm rests on top of a sugar cone that is 12 cm deep and has a diameter of 5 cm. If all of the ice cream melts into the cone, what percent of the cone will be filled? Round to the nearest percent. 83%

22. Writing A cylinder, a cone, and a sphere have the dimensions indicated in the diagram below.

- a. What are the formulas for the volume of the cone and the volume of the cylinder in terms of \( r \)? Give each answer in terms of \( \pi \).
  - cone: \( V = \frac{1}{3}\pi r^2 \); cylinder: \( V = \pi r^3 \)

- b. If \( r = 9 \text{ in.} \), what are the volumes of the cone, cylinder, and sphere? 243\( \pi \), 729\( \pi \), 972\( \pi \)

- c. How are the volumes related? The volume of a sphere is equal to the sum of the volumes of the cone and the cylinder.

- d. How can you show that this relationship is true for all values of \( r \)?
  - by using the formulas for the volumes to show that \( \frac{1}{3}\pi r^3 + \pi r^3 = \frac{4}{3}\pi r^3 \)

23. A bowling ball must have a diameter of 8.5 in. If the bowling ball weighs 16 lb, find the density (lb/in.\(^3\)) of the bowling ball. Density is the quotient of weight divided by volume. Round your answer to the nearest hundredth. 0.05 lb/in.\(^3\)

24. Open-Ended Draw two spheres such that the volume of one sphere is eight times the volume of the other sphere. Answers may vary. The larger sphere’s radius will be twice the radius of the smaller sphere.
11-6 Standardized Test Prep
Surface Areas and Volumes of Spheres

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. What is the approximate volume of the sphere? A
   - A 524 m³
   - B 1000 m³
   - C 1256 m³
   - D 1570 m³

2. What is the approximate surface area of the sphere? G
   - F 225 yd²
   - H 1767 yd²
   - G 707 yd²
   - I 5301 yd²

3. What is the approximate volume of the sphere if the surface area is 482.8 mm²? A
   - A 998 mm³
   - B 1126 mm³
   - C 2042 mm³
   - D 2993 mm³

4. What is the approximate surface area of the sphere? G
   - F 342.3 km²
   - H 903.4 km²
   - G 451.9 km²
   - I 2713 km²

5. What is the approximate radius of a sphere whose volume is 1349 cm³? B
   - A 5.7 cm
   - B 6.9 cm
   - C 11 cm
   - D 14.7 cm

Short Response

6. Suppose a wealthy entrepreneur commissions the design of a spherical spaceship to house a small group for a week in orbit around the Earth. The designer allocates 1000 ft³ for each person, plus an additional 4073.5 ft³ for various necessary machines. As in a recreational vehicle, the personal space is largely occupied by items such as beds, shower and toilet facilities, and a kitchenette. The diameter of the ship is 26.8 ft. What is the volume of the spaceship, and for approximately how many people is the ship designed?

[2] V = 10,078.7 ft³; 6 people
[1] incorrect volume or incorrect number of people
[0] no response or incorrect response

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11-6 Enrichment
Surface Areas and Volumes of Spheres

Two spheres of radius \( R \) and \( r \) are concentric if the centers of both spheres are the same point. Assume that \( r > R \).

1. What is the volume of the outer (larger) sphere? \( \frac{4}{3}\pi r^3 \)

2. What is the volume of the inner (smaller) sphere? \( \frac{4}{3}\pi R^3 \)

3. What is the volume of the space between the smaller and the larger sphere? \( \frac{4}{3}\pi (r^3 - R^3) \)

The radius of Earth is approximately 4000 mi, and the radius of its core, which is a sphere of molten metals, is about 800 mi.

4. What is the volume of Earth that lies outside the core to the nearest billion mi\(^3\)? \( 266,000,000,000 \) mi\(^3\)

5. Use \((x^3 - y^3) = (x - y)(x^2 + xy + y^2)\) to factor the expression for the volume lying between two concentric spheres. \( \frac{4}{3}\pi (r - R)(r^2 + rR + R^2) \)

The thickness \( d \) of the thin spherical shell representing the difference between the two spheres is defined to be \( d = r - R \).

6. Using \( r \approx R_c \) compute the approximate volume of a thin spherical shell of radius \( R \) and thickness \( d \). \( \frac{4}{3}\pi d(3R^2) = 4\pi dR^2 \)

7. To see the accuracy of this approximation, fill in the table to two decimal places.

<table>
<thead>
<tr>
<th>( R )</th>
<th>( d )</th>
<th>Exact Volume of Shell</th>
<th>Approximate Volume of Shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.1</td>
<td>126.92</td>
<td>125.66</td>
</tr>
<tr>
<td>50</td>
<td>1</td>
<td>32,048.43</td>
<td>31,415.93</td>
</tr>
<tr>
<td>100</td>
<td>3</td>
<td>388,413.95</td>
<td>376,991.12</td>
</tr>
</tbody>
</table>

Most golf balls consist of three concentric spheres: a liquid-filled center, elastic windings, and a dimpled cover. The size and materials of these three components vary for each type of golf ball. A specific type of golf ball has radius 0.84375 in. The center of the golf ball has radius 0.565 in., and the cover has thickness 0.05 in.

8. What is the volume of the outside cover to the nearest thousandth? \( 0.421 \) in.\(^3\)

9. What is the volume of the elastic winding to the nearest hundreth? \( 1.34 \) in.\(^3\)
11-6 Reteaching
Surface Areas and Volumes of Spheres

Problem

What are the surface area and volume of the sphere?
Substitute $r = 5$ into each formula, and simplify.

\[
\text{S.A.} = 4\pi r^2 \quad \text{V} = \frac{4}{3}\pi r^3 \\
= 4\pi(5)^2 \quad = \frac{4}{3}\pi(5)^3 \\
= 100\pi \quad \approx \frac{500\pi}{3} \\
\approx 314.2 \quad \approx 523.6
\]

The surface area of the sphere is about 314.2 in.\(^2\). The volume of the sphere is about 523.6 in.\(^3\).

Exercises

Use the figures at the right to guide you in completing the following.

1. Use a compass to draw two circles, each with radius 3 in. Cut out each circle. Check students’ work.

2. Fold one circle in half three successive times. Number the central angles 1 through 8. Check students’ work.

3. Cut out the sectors, and tape them together as shown. Check students’ work.

4. Take the other circle, fold it in half, and tape it to the rearranged circle so that they form a quadrant of a sphere. Check students’ work.

5. The area of one circle has covered one quadrant of a sphere. How many circles would cover the entire sphere? four circles

6. How is the radius of the sphere related to the radius of the circle? They are the same.

Find the volume and surface area of a sphere with the given radius or diameter. Round your answers to the nearest tenth.

7. 
\[
\text{V} = 523.6 \text{ in.}^3; \\
\text{S.A.} = 314.2 \text{ in.}^2
\]

8. 
\[
\text{V} = 113.1 \text{ cm}^3; \\
\text{S.A.} = 113.1 \text{ cm}^2
\]

9. 
\[
\text{V} = 7,238.2 \text{ m}^3; \\
\text{S.A.} = 1,809.6 \text{ m}^2
\]
Find the volume and surface area of the sphere. Round to the nearest tenth.

10. \[ S.A. = 2463.0 \text{ in}^2; \quad V = 11,494.0 \text{ in}^3 \]
11. \[ S.A. = 6,157,521.6 \text{ m}^2; \quad V = 1,436,755,040.2 \text{ m}^3 \]
12. \[ S.A. = 12.6 \text{ cm}^2; \quad V = 4.2 \text{ cm}^3 \]
13. \[ S.A. = 1256.6 \text{ m}^2; \quad V = 4188.8 \text{ m}^3 \]
14. \[ S.A. = 50.3 \text{ ft}^2; \quad V = 33.5 \text{ ft}^3 \]
15. \[ S.A. = 153.9 \text{ m}^2; \quad V = 179.6 \text{ m}^3 \]

A sphere has the volume given. Find its surface area to the nearest whole number.

16. \[ V = 1436.8 \text{ m}^3 \quad S.A. = 616 \text{ m}^2 \]
17. \[ V = 808 \text{ cm}^3 \quad S.A. = 420 \text{ cm}^2 \]
18. \[ V = 72 \text{ m}^3 \quad S.A. = 84 \text{ m}^2 \]

Find the volume of each sphere with the given surface area. Round to the nearest whole number.

19. \[ S.A. = 435 \text{ yd}^2 \quad V = 853 \text{ yd}^3 \]
20. \[ S.A. = 907 \text{ cm}^2 \quad V = 2569 \text{ cm}^3 \]
21. \[ S.A. = 28 \text{ m}^2 \quad V = 14 \text{ m}^3 \]

22. **Visualization** The region enclosed by the semicircle at the right is revolved completely about the \(x\)-axis.
   a. Describe the solid of revolution that is formed. a sphere
   b. Find its volume in terms of \(\pi\). \(85\frac{1}{3}\pi\) units\(^3\)
   c. Find its surface area in terms of \(\pi\). \(64\pi\) units\(^2\)

23. The sphere at the right fits snugly inside a cube with 18 cm edges. What is the volume of the sphere? What is the surface area of the sphere? Leave your answers in terms of \(\pi\). \(972\pi\) cm\(^3\); \(324\pi\) cm\(^2\)
11-7 Additional Vocabulary Support
Areas and Volumes of Similar Solids

Choose the word from the list below that best matches each description.

<table>
<thead>
<tr>
<th>area</th>
<th>lateral area</th>
<th>linear dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>proportion</td>
<td>ratio</td>
<td>scale factor</td>
</tr>
<tr>
<td>similar</td>
<td>surface area</td>
<td>volume</td>
</tr>
</tbody>
</table>

1. a comparison of two quantities  
   ___________ ratio ___________

2. figures that are the same shape and have corresponding sides that are proportional  
   ___________ similar ___________

3. equal ratios  
   ___________ proportion ___________

4. the ratio of the corresponding dimensions of similar figures  
   ___________ scale factor ___________

Use a word from the list above to complete each sentence.

5. The sum of the areas of each face of a solid figure that is not a base is the ___________ lateral area ___________.

6. To find the ___________ volume ___________ of a rectangular prism, multiply the area of the base by the height.

7. A solid’s length, width, and height are its ___________ linear dimensions ___________.

8. The sum of the areas of each face of a solid figure is its ___________ surface area ___________.

Circle the correct value for the figures at the right.

9. surface area of figure A  
   [400 ft², 448 ft²]  
   ___________ 448 ft² ___________

10. volume of figure B  
    [60 ft³, 112 ft³]  
    ___________ 112 ft³ ___________

11. scale factor of figure A to figure B  
    [2 : 1, 3 : 1, 10 : 1]  
    ___________ 2 : 1 ___________

Multiple Choice

12. If the scale factor between two figures is 2 : 1, what is the ratio of the volume of the figures?  
    ___________ D ___________

    [A] 1 : 3  
    [B] 3 : 1  
    [C] 4 : 1  
    [D] 8 : 1
11-7 Think About a Plan
Areas and Volumes of Similar Solids

Reasoning A carpenter is making a blanket chest based on an antique chest. Both chests have the shape of a rectangular prism. The length, width, and height of the new chest will all be 4 in. greater than the respective dimensions of the antique. Will the chests be similar? Explain.

Understanding the Problem

1. How much longer will the blanket chest be than the antique chest? How much wider will it be? How much taller will it be? Draw a diagram of the blanket chest showing the increase in each dimension.

The new chest will be 4 in. longer, wider, and taller than the antique chest.

2. If two solids are similar, what must be true about their corresponding dimensions?

Each pair of corresponding dimensions must satisfy the same ratio.

Planning the Solution

3. Practice explaining your reasoning. Imagine a chest has dimensions 3 ft-by-3 ft-by-3 ft. Add 4 in. to each dimension. Is the new chest similar to the old chest? Explain.

Yes; the ratios between each pair of corresponding dimensions are equal.

4. Practice explaining your reasoning. Suppose a chest has dimensions 4 ft-by-3 ft-by-3 ft. Add 4 in. to each dimension. Is the new chest similar to the old chest? Explain.

No; the new chest is not similar because the ratios between each pair of corresponding dimensions are not equal.

Getting an Answer

5. How can you generalize the dimensions of the old chest?

Sample: The dimensions can be represented by the variables \( l, w, \) and \( h. \)

6. Using the dimensions of the chest, determine whether the new chest and the old chest will be similar.

Unless \( l = w = h, \) then the ratios between each pair of corresponding dimensions will not be equal, and they will not be similar.
Are the two figures similar? If so, give the scale factor of the first figure to the second figure.

1. yes; 5 : 2
2. yes; 3 : 5
3. no
4. no

5. two cubes, one with 5-in. edges, the other with 6-in. edges yes; 5 : 6
6. a cylinder and a cone, each with 6-m radii and 4-m heights no

Each pair of figures is similar. Use the given information to find the scale factor of the smaller figure to the larger figure.

7. 4 : 5
8. 5 : 6
9. 5 : 7
10. 6 : 11

The surface areas of two similar figures are given. The volume of the larger figure is given. Find the volume of the smaller figure.

11. S.A. = 36 m²  48 m³
    S.A. = 225 m²  750 m³
12. S.A. = 108 in.²  594 in.³
    S.A. = 192 in.²  1408 in.³
13. S.A. = 49 m²  16 m³
    S.A. = 441 m²  432 m³

14. A shipping box holds 350 golf balls. A larger shipping box has dimensions triple the size of the other box. How many golf balls does the larger box hold? 9450
The volumes of two similar figures are given. The surface area of the smaller figure is given. Find the surface area of the larger figure.

15. \( V = 8 \text{ m}^3 \)  
   \( V = 27 \text{ m}^3 \)  
   \( \text{S.A.} = 36 \text{ m}^2 \)  
   \( 81 \text{ m}^2 \)

16. \( V = 125 \text{ in.}^3 \)  
   \( V = 216 \text{ in.}^3 \)  
   \( \text{S.A.} = 200 \text{ in.}^2 \)  
   \( 288 \text{ in.}^2 \)

17. \( V = 3 \text{ ft}^3 \)  
   \( V = 375 \text{ ft}^3 \)  
   \( \text{S.A.} = 4 \text{ ft}^2 \)  
   \( 100 \text{ ft}^2 \)

18. A cylindrical thermos has a radius of 2 in. and is 5 in. high. It holds 10 fl oz. To the nearest ounce, how many ounces will a similar thermos with a radius of 3 in. hold? \( 34 \text{ fl oz} \)

19. **Compare and Contrast**  You have a set of three similar nesting gift boxes.
   Each box is a regular hexagonal prism. The large box has 10-cm base edges.
   The medium box has 6-cm base edges. The small box has 3-cm base edges.
   How does the volume of each box compare to every other box? **The ratio of the volumes of the large box to the medium box is 125 : 27; the ratio of the volumes of the large box to the small box is 1000 : 27; the ratio of the volumes of the medium box to the small box is 8 : 1.**

20. Two similar pyramids have heights 6 m and 9 m.
   a. What is their scale factor? \( 2 : 3 \)
   b. What is the ratio of their surface areas? \( 4 : 9 \)
   c. What is the ratio of their volumes? \( 8 : 27 \)

21. A small, spherical hamster ball has a diameter of 8 in. and a volume of about 268 in.\(^3\). A larger ball has a diameter of 14 in. Estimate the volume of the larger hamster ball. \( 1436 \text{ in.}^3 \)

22. **Error Analysis**  A classmate says that a rectangular prism that is 6 cm long, 8 cm wide, and 15 cm high is similar to a rectangular prism that is 12 cm long, 14 cm wide, and 21 cm high. Explain your classmate’s error. **The ratios of the dimensions are not the same: the ratio of the lengths is 1 : 2, the ratio of the widths is 4 : 7, and the ratio of the heights is 5 : 7. Each dimension of the larger prism is 6 cm greater.**

23. The lateral area of two similar cylinders is 64 m\(^2\) and 144 m\(^2\). The volume of the larger cylinder is 216 m\(^3\). What is the volume of the smaller cylinder? \( 64 \text{ m}^3 \)

24. The volumes of two similar prisms are 135 ft\(^3\) and 5000 ft\(^3\).
   a. Find the ratio of their heights. \( 3 : 10 \)
   b. Find the ratio of the area of their bases. \( 9 : 100 \)
Are the two figures similar? If so, give the scale factor of the first figure to the second figure.

1. yes; 2 : 3

2. yes; 4 : 5

3. no

5. two cubes, one with 6-in. edges, the other with 8-in. edges yes; 3 : 4

6. a cylinder and a cone, each with 9-m radii and 5-m heights no

Each pair of figures is similar. Use the given information to find the scale factor of the smaller figure to the larger figure.

7. To start, write a proportion using the ratio of the volumes of the solids.

\[ \frac{a^3}{b^3} = \frac{64}{216} \]

8. 4 : 7

9. 3 : 5

10. Two similar cones have heights 4 m and 12 m.
   a. What is their scale factor? 1 : 3
   b. What is the ratio of their surface areas? 1 : 9
   c. What is the ratio of their volumes? 1 : 27

11. A shipping box holds 450 golf balls. A larger shipping box has dimensions triple the size of the other box. How many golf balls does the larger box hold? 12,150 balls
11-7 Practice (continued)  Form K

Areas and Volumes of Similar Solids

The surface areas of two similar figures are given. The volume of the larger figure is given. Find the volume of the smaller figure.

12. S.A. = 94 m²
   S.A. = 846 m²
   V = 1620 m³
   60 m³
   To start, find the scale factor \( a : b \).
   \[
   \frac{a^2}{b^2} = \frac{94}{846}
   \]

13. S.A. = 240 m²
   S.A. = 1500 m²
   V = 1562.5 m³
   100 m³

14. S.A. = 96 in.²
   S.A. = 216 in.²
   V = 216 in.³
   64 in.³

The volumes of two similar figures are given. The surface area of the larger figure is given. Find the surface area of the smaller figure.

15. V = 384 m³
   V = 10,368 m³
   S.A. = 3168 m² 352 m²

16. V = 216 in.³
   V = 1728 in.³
   S.A. = 864 in.² 216 in.²

17. A cylindrical thermos has a radius of 3 in. and is 12 in. high. It holds 20 fl oz. To the nearest ounce, how many ounces will a similar thermos with a radius of 4 in. hold? 47 fl oz

18. You have a set of three similar gift boxes. Each box is a rectangular prism. The large box has 15-cm base edges. The medium box has 10-cm base edges. The small box has 5-cm base edges. How does the volume of each box compare to every other box? The ratio of the volume of the large box to that of the medium box is 27 : 8; the ratio of the volume of the large box to that of the small box is 27 : 1; the ratio of the volume of the medium box to that of the small box is 8 : 1.

19. A baseball and a softball are similar in shape. The baseball has a radius of 1.25 in. and a volume of 8.18 in.³. If the volume of a softball is 65.44 in.³, what is the radius of the softball? 2.5 in.

20. Error Analysis A classmate says that a rectangular prism that is 9 cm long, 12 cm wide, and 15 cm high is similar to a rectangular prism that is 12 cm long, 16 cm wide, and 21 cm high. Explain your classmate’s error. The ratios of the dimensions are not the same. The ratio of the lengths is 3 : 4, the ratio of the widths is 3 : 4, but the ratio of the heights is 5 : 7.

21. The volumes of two similar prisms are 512 ft³ and 8000 ft³.
   a. Find the ratio of their heights. 2 : 5
   b. Find the ratio of the area of their bases. 4 : 25
Multiple Choice

For Exercises 1–5, choose the correct letter.

1. Which of the figures shown below are similar? D

2. The measure of the side of a cube is 6 ft. The measure of the side of a second cube is 18 ft. What is the scale factor of the cubes? G
   - F 1 : 2
   - G 1 : 3
   - H 1 : 9
   - I 1 : 27

3. What is the ratio of the surface areas of the similar square pyramids at the right? C
   - A 4 : 5
   - B 8 : 10
   - C 16 : 25
   - D 64 : 125

4. What is the ratio of the volumes of the similar square pyramids above? I
   - F 4 : 5
   - G 8 : 10
   - H 16 : 25
   - I 64 : 125

5. The surface areas of two similar triangular prisms are 132 m\(^2\) and 297 m\(^2\). The volume of the smaller prism is 264 m\(^3\). What is the volume of the larger prism? B
   - A 594 m\(^3\)
   - B 891 m\(^3\)
   - C 1336.5 m\(^3\)
   - D 3007.125 m\(^3\)

Short Response

6. A medium-sized box can hold 55 T-shirts. If the dimensions of a jumbo box are three times that of the medium box, how many T-shirts can the jumbo box hold? Explain. [2] 1485 T-shirts; if the dimensions of the jumbo box are three times those of the medium box, the scale factor is 1 : 3. The ratio of the volumes is 1\(^3\) : 3\(^3\), or 1 : 27. Set up a proportion to solve: \(\frac{1}{27} = \frac{55}{x}\); \(x = 27 \cdot 55 = 1485\). [1] error in calculation, or work not shown [0] incorrect or no response
Use each situation to complete the exercises that follow. Any costs given are proportional to the number of units.

Two storage bins are built in the form of rectangular prisms, and the two bins are similar. One stores wheat at a cost of $0.15 per bushel, and the other stores corn at a cost of $0.20 per bushel. The bin storing the wheat has a square base 80 ft on a side and is 120 ft tall.

1. If the cost of storing the wheat is $8000, and the cost of storing the corn is $36,000, find the height and the length to the nearest whole number of a side of the base of the bin storing corn. 180 ft; 120 ft

A sphere with a 4-in. radius is being plated with silver for a cost of $30.

2. If gold plating costs 75% more than silver plating, how much will it cost to gold-plate a sphere with radius 6 in.? $118.13

3. To the nearest hundredth, what is the radius of a sphere that is plated with silver for a cost of $80? 6.53 in.

4. To the nearest hundredth, what is the radius of a sphere that is plated with gold for a cost of $100? 5.52 in.

Two similar cylinders contain juice. The first cylinder has radius 6 in. and height 10 in., contains orange juice, and sells for $2.40.

5. If grapefruit juice costs two-thirds of the price of an equal volume of orange juice, what is the cost of a container of grapefruit juice that has radius 9 in. and height 15 in.? $5.40

6. To the nearest hundredth, what is the radius of a similar container holding $4.00 worth of grapefruit juice? 8.14 in.

7. To the nearest hundredth, what is its height? 13.57 in.

Two similar cones have a combined volume of 400 in.$^3$, and the larger cone holds 80 in.$^3$ more than the smaller cone, which has a radius of 3 in.

8. To the nearest hundredth, what is the height of the smaller cone? 16.98 in.

9. What is the radius of the larger cone? 3.43 in.

10. What is the height of the larger cone? 19.43 in.
11-7 Reteaching
Areas and Volumes of Similar Solids

When two solids are similar, their corresponding dimensions are proportional.

Rectangular prisms \( A \) and \( B \) are similar because the ratio of their corresponding dimensions is \( \frac{2}{3} \).

- height: \( \frac{8 \text{ m}}{12 \text{ m}} = \frac{2}{3} \)
- length: \( \frac{2 \text{ m}}{3 \text{ m}} = \frac{2}{3} \)
- width: \( \frac{4 \text{ m}}{6 \text{ m}} = \frac{2}{3} \)

The ratio of the corresponding dimensions of similar solids is called the scale factor. All the linear dimensions (length, width, and height) of a solid must have the same scale factor for the solids to be similar.

**Areas and Volumes of Similar Solids**

**Area**
- The ratio of corresponding areas of similar solids is the square of the scale factor.
- The ratio of the areas of prisms \( A \) and \( B \) is \( \frac{2^2}{3^2}, \) or \( \frac{4}{9} \).

**Volume**
- The ratio of the volumes of similar solids is the cube of the scale factor.
- The ratio of the volumes of prisms \( A \) and \( B \) is \( \frac{2^3}{3^3}, \) or \( \frac{8}{27} \).

**Problem**

The pyramids shown are similar, and they have volumes of 216 in.\(^3\) and 125 in.\(^3\). The larger pyramid has surface area 250 in.\(^2\).

What is the ratio of their surface areas?
What is the surface area of the smaller pyramid?

By Theorem 11-12, if similar solids have similarity ratio \( a : b \), then the ratio of their volumes is \( a^3 : b^3 \).

So,

\[
\frac{a^3}{b^3} = \frac{216}{125}
\]

\[
\frac{a}{b} = \frac{6}{5}
\]

Take the cube root of both sides to get \( a : b \).

\[
\frac{a^2}{b^2} = \frac{36}{25}
\]

Square both sides to get \( a^2 : b^2 \).

Ratio of surface areas = 36 : 25

If the larger pyramid has surface area 250 in.\(^2\), let the smaller pyramid have surface area \( x \).

Then,

\[
\frac{250}{x} = \frac{36}{25}
\]

\[36x = 6250\]

\[x \approx 173.6 \text{ in.}^2\]

The surface area of the smaller pyramid is about 173.6 in.\(^2\).
**Exercises**

Find the scale factors.

1. Similar cylinders have volumes of $200\pi$ in.$^3$ and $25\pi$ in.$^3$.  $2:1$

2. Similar cylinders have surface areas of $45\pi$ in.$^2$ and $20\pi$ in.$^2$.  $3:2$

Are the two figures similar? If so, give the scale factor.

3.  yes; $1:2$

4.  no

Each pair of figures is similar. Use the given information to find the scale factor of the smaller figure to the larger figure.

5.  $V = 135\pi$ in.$^3$  $V = 320\pi$ in.$^3$  $3:4$

6.  S.A. = $32\text{ cm}^2$  S.A. = $162\text{ cm}^2$  $4:9$

Find the ratio of volumes.

7. Two cubes have sides of length 4 cm and 5 cm.  $64:125$

8. Two cubes have surface areas of 64 in.$^2$ and 49 in.$^2$.  $512:343$

The surface areas of two similar figures are given. The volume of the larger figure is given. Find the volume of the smaller figure.

9. S.A. = $16\text{ cm}^2$  $32\text{ cm}^3$  S.A. = $100\text{ cm}^2$  S.A. = $294\text{ ft}^2$  S.A. = $500\text{ cm}^3$  S.A. = $3430\text{ ft}^3$  S.A. = $80\text{ m}^2$  S.A. = $135\text{ m}^3$

The volumes of two similar figures are given. The surface area of the smaller figure is given. Find the surface area of the larger figure.

10. $V = 12\text{ in.}^3$  $48\text{ in.}^2$  $V = 6\text{ cm}^3$  $96\text{ cm}^2$  $V = 40\text{ ft}^3$  $45\text{ ft}^2$

11. $V = 96\text{ in.}^3$  $V = 384\text{ cm}^3$  $V = 135\text{ ft}^3$  $S.A. = 6\text{ cm}^2$  $S.A. = 6\text{ ft}^2$  $S.A. = 20\text{ ft}^2$